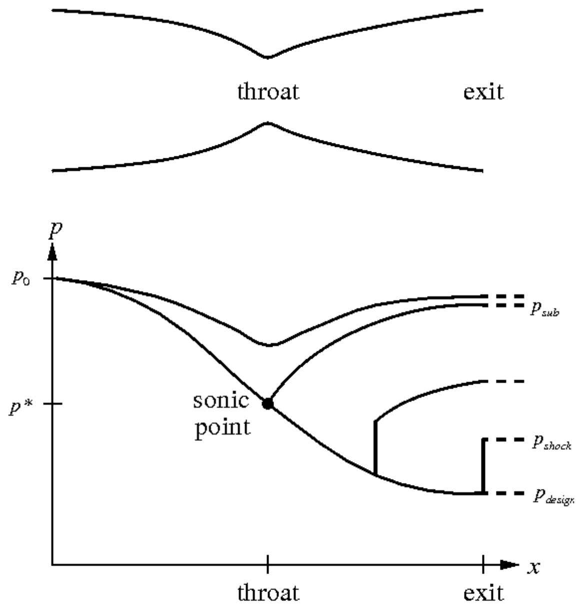
Converging-Diverging Nozzle



The behaviour is governed by the ratio of back pressure p_{back} to total pressure p_0.

For pressures above p_{sub} the flow remains subsonic throughout.

For pressures below p_{sub} the flow goes supersonic at the throat; the throat is choked and the maximum mass flow rate is achieved.

At the design pressure p_{design} the flow passes smoothly from subsonic to supersonic without shocks.

To determine p_{sub} and p_{design}, since $A^* = A_{throat}$, use the subsonic and supersonic Mach numbers corresponding to isentropic flow with area ratio A_{exit}/A_{throat}:

$$\frac{p_{sub}}{p_0} = \frac{1}{\left[1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}_{sub}^2\right]^{\gamma/(\gamma-1)}}$$

$$\frac{p_{design}}{p_0} = \frac{1}{\left[1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}_{design}^2\right]^{\gamma/(\gamma-1)}}$$

where Ma_{sub} and Ma_{design} are the subsonic and supersonic solutions, respectively, of

$$\frac{A_{exit}}{A_{throat}} = \frac{1}{\mathrm{Ma}}\left[\frac{1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}^2}{\tfrac{1}{2}(\gamma + 1)}\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

For back pressures between p_{sub} and p_{design} there are shocks inside the nozzle or in the exit jet. At $p_{shock\text{-}exit}$ a shock occurs at the exit plane. This value can be computed by assuming a normal shock with upstream values p_{design} and Ma_{design} and downstream pressure $p_{shock\text{-}exit}$:

$$\frac{p_{shock-exit}}{p_{design}} = \frac{\gamma \mathrm{Ma}_{design}^2 - \tfrac{1}{2}(\gamma - 1)}{\tfrac{1}{2}(\gamma + 1)}$$

The maximum mass flow rate occurs when the throat is sonic:

$$\dot{m}_{\max} = \frac{p_0 A_{throat}}{\sqrt{RT_0}}\gamma^{1/2}\left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

<u>Nomenclature</u>

- p = pressure;
- p_0 = total pressure;
- Ma = Mach number;
- γ = ratio of specific heat capacities;
- A = cross-sectional area;
- $\dot{m}$ = mass flow rate.