1-D Gas Flow

Ideal gas law:

$$p = \rho RT$$

For isentropic changes:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Speed of sound:

$$c = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$$

Adiabatic flow:

$$\frac{T_0}{T} = 1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}^2$$

If isentropic:

$$\frac{p_0}{p} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

Sonic values (*) are found by setting Ma = 1:

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2}, \qquad \frac{T}{T^*} = \frac{\tfrac{1}{2}(\gamma + 1)}{1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}^2}$$

with corresponding isentropic formulae for p and ρ.

The ratio of local cross-sectional area to the area at sonic conditions is given by:

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\frac{1 + \tfrac{1}{2}(\gamma - 1)\mathrm{Ma}^2}{\tfrac{1}{2}(\gamma + 1)}\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

Maximum mass flow occurs at the sonic area and is given by:

$$\dot{m}_{\max} = \frac{p_0 A^*}{\sqrt{RT_0}}\gamma^{1/2}\left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

<u>Nomenclature</u>

p = pressure;
ρ = density;
T = absolute temperature;
u = velocity;
c = sound speed;
Ma = u/c = Mach number;
c_p, c_v = specific heats capacities at constant pressure and volume, respectively;
γ = ratio of specific heat capacities;
R = gas constant, given by R_0/m, where R_0 is universal gas constant, m is molar mass;
p_0, ρ_0, T_0 = total pressure, density, temperature;
A = cross-sectional area.