Bed-Load Sediment Transport Formulae

Dimensionless Quantities

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}} \qquad \text{(dimensionless bed flux)}$$

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd} \qquad \text{(dimensionless shear stress; or } \textit{Shields stress}\text{)}$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \qquad \text{(dimensionless particle diameter)}$$

Bed-Load Transport Formulae

Reference	Formula	Comments
Meyer-Peter and Muller (1948)	$q^* = 8(\tau^* - \tau^*_{crit})^{3/2}$	The original paper assumed $\tau^*_{crit} = 0.047$
Einstein-Brown (Brown, 1950)	$q^* = \begin{cases} \dfrac{K\exp(-0.391/\tau^*)}{0.465} & \tau^* < 0.182 \\ 40K{\tau^*}^3 & \tau^* \geq 0.182 \end{cases}$	$K = \sqrt{\frac{2}{3} + \frac{36}{{d^*}^3}} - \sqrt{\frac{36}{{d^*}^3}}$
Yalin (1963)	$q^* = 0.635r\sqrt{\tau^*}[1 - \frac{1}{\sigma r}\ln(1+\sigma r)]$	$r = \frac{\tau^*}{\tau^*_{crit}} - 1,$ $\sigma = 2.45\frac{\sqrt{\tau^*_{crit}}}{(\rho_s/\rho)^{0.4}}$
Van Rijn (1984)	$q^* = \frac{0.053}{{d^*}^{0.3}}(\frac{\tau^*}{\tau^*_{crit}} - 1)^{2.1}$	His equation 22. There are some variants and restrictions on parameters.
Nielsen (1992)	$q^* = 12(\tau^* - \tau^*_{crit})\sqrt{\tau^*}$	

Critical Shear Stress

Van Rijn (1984):

$$\tau^*_{crit} = \begin{cases} 0.24{d^*}^{-1} & d^* \leq 4 \\ 0.14{d^*}^{-0.64} & 4 < d^* \leq 10 \\ 0.04{d^*}^{-0.1} & 10 < d^* \leq 20 \\ 0.013{d^*}^{0.29} & 20 < d^* \leq 150 \\ 0.056 & d^* > 150 \end{cases}$$

Soulsby (1997):

$$\tau^*_{crit} = \frac{0.30}{1+1.2d^*} + 0.055\,[1 - \exp(-0.020d^*)]$$

Means of Estimating the Bed Shear Stress

(i) Normal-flow relationship

$\tau = \rho g R_h S$ ($R_h = h$ for a wide channel.)

(ii) Assumed log-law velocity profile

$$\tau = \rho u_\tau^2, \qquad u_\tau = \frac{\kappa U_{av}}{\ln(\frac{h}{z_0}) - 1}$$

Variables

q_b = bed flux (volume rate of transport per unit length of surface)
τ_b = bed shear stress
d = particle diameter
ρ_s = sediment density
ρ = fluid density
s (= ρ_s/ρ) = relative density
g = acceleration due to gravity
ν = kinematic viscosity
R_h = hydraulic radius
h = water depth
S = channel slope
u_τ = friction velocity (defined by $\tau_b = \rho u_\tau^2$)
z_0 = roughness length
κ = von Karman constant (0.41)
U_{av} = average velocity

References

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