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## Lecture 3: Region Based Vision

Dr Carole Twining  
Thursday 18th March  
1:00pm – 1:50pm

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Slide 2

## Segmenting an Image

Assigning labels to pixels (cat, ball, floor)

- Point processing:
  - colour or grayscale values, thresholding
- Neighbourhood Processing:
  - Regions of similar colours or textures
- Edge information (next lecture)
- Prior information: (model-based vision)
  - I know what I *expect* a cat to look like

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Slide 3

## Overview

- Automatic threshold detection
  - Earlier, we did by inspection/guessing
- Multi-Spectral segmentation
  - satellite and medical image data
- Split and Merge
  - Hierarchical, region-based approach
- Relaxation labelling
  - probabilistic, learning approach
- Segmentation as optimisation

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## Automatic Threshold Selection

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## Automatic Thresholding: Optimal Segmentation Rule

Image Histogram

Assume scene mixture of substances, each with normal/gaussian distribution of possible image values

- Minimum error in probabilistic terms
- But sum of gaussians not easy to find
- Doesn't always fit actual distribution

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## Automatic Thresholding: Otsu's Method

# of pixels  $n(i)$

Mean across purples:  

$$\bar{i}_P = \frac{1}{N_P} \sum_{i=0}^T i \times n(i)$$
 Variance for purples:  

$$\sigma_P^2 = \frac{1}{N_P} \sum_{i=0}^T n(i) [i - \bar{i}_P]^2$$

Choose  $T$  to minimize:  

$$N_P \sigma_P^2 + N_G \sigma_G^2$$

- Extend to multiple classes

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## Automatic Thresholding: Max Entropy

For two sub-populations:

$$p_P(i) = \frac{n(i)}{N_P}, \quad i < T,$$

$$p_G(i) = \frac{n(i)}{N_G}, \quad i \geq T.$$

Entropy:  $-\sum p \ln p$

Two Entropies:  

$$H_P = -\sum_{i < T} p_P(i) \ln p_P(i) \quad \& \quad H_G = -\sum_{i \geq T} p_G(i) \ln p_G(i)$$

Minimise:  $H_G + H_P$  to find  $T$ .

- Makes two sub-populations as peaky as possible

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## Automatic Thresholding: Example

combine

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## Automatic Thresholding: Summary

- Geometric shape of histogram (bumps, curves etc)
  - Algorithm or just by inspection
- Statistics of sub-populations
  - Otsu & variance
  - Entropy methods
- Model-based methods:
  - Mixture of gaussians
- And so on. > 40 methods surveyed in literature
- Fundamental limit on effectiveness:
  - Never give great result if distributions overlap
- Whatever method, need further processing

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## Multi-Spectral Segmentation

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## Multi-spectral Segmentation

- Multiple measurements at each pixel:
  - Satellite remote imaging, various wavebands
  - MR imaging, various imaging sequences
  - Colour (RGB channels, HSB etc)
- Scattergram of pixels in vector space:
  - Can't separate using single measurement
  - Can using multiple

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## Multi-Spectral Segmentation: Example

Spectral Bands

Over-ripe Orange

Scratched Orange

Multispectral Image Segmentation by Energy Minimization for Fruit Quality Estimation: *Martínez-Uso, Pla, and García-Sevilla, Pattern Recognition and Image Analysis, 2005*

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# Split and Merge

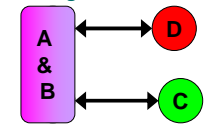
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## Split and Merge

- Obvious approaches to segmentation:
  - Start from small regions and stitch them together } **Combine**
  - Start from large regions and split them
- Start with large regions, **split** non-uniform regions
  - e.g. variance  $\sigma^2 >$  threshold
- **Merge** similar adjacent regions
  - e.g. combined variance  $\sigma^2 <$  threshold
- Region adjacency graph
  - housekeeping for adjacency as regions become irregular
  - regions are nodes, adjacency relations arcs
  - simple update rules during splitting and merging

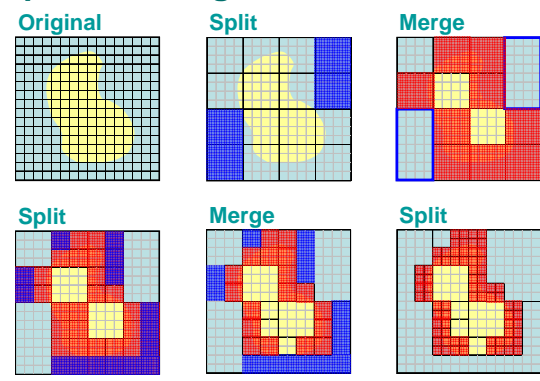


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## Split and Merge

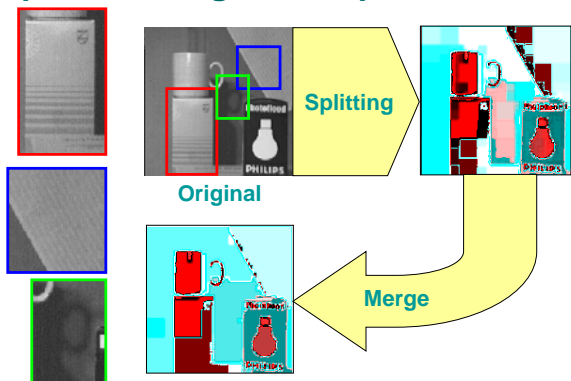


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## Split and Merge: Example



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# Relaxation Labelling

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## Aside: Conditional Probability

probability of A given that B is the case

$$P(A | B)$$

- $P(\text{pet}) = \frac{(\text{fish} + \text{dog} + \text{cat})}{\text{ALL}}$  etc
- $P(\text{pet} | \text{mammal}) = \frac{(\text{dog} + \text{cat})}{(\text{dog} + \text{cat} + \text{whale})}$
- $P(\text{mammal} | \text{pet}) = \frac{(\text{dog} + \text{cat})}{(\text{fish} + \text{dog} + \text{cat})}$
- Bayes Theorem:

$$\frac{(\text{dog} + \text{cat})}{(\text{dog} + \text{cat} + \text{whale})} \times \frac{(\text{fish} + \text{dog} + \text{cat})}{\text{ALL}} = \frac{(\text{fish} + \text{dog} + \text{cat})}{(\text{fish} + \text{dog} + \text{cat})} \times \frac{(\text{dog} + \text{cat})}{\text{ALL}}$$

- $P(\text{pet} | \text{mammal})P(\text{mammal}) = P(\text{mammal} | \text{pet})P(\text{pet})$

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## Relaxation Labelling:

- Image histogram, object/background

Overlap: mistakes in labelling

Values from object pixels

threshold

Values from background

Label assignments

**Context:**

**Context to resolve ambiguity**

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## Relaxation Labelling

- Evidence for a label at a pixel:
  - Measurements at that pixel (e.g., pixel value)
  - Context for that pixel (i.e., what neighbours are doing)
- Iterative approach, labelling evolves
- Soft-assignment of labels:
  - Possible labels:  $\{l_\mu : \mu = 1, \dots, n\}$
  - $P_i(l_\mu)$  : Probability that pixel  $i$  has label  $l_\mu$ .
  - $\sum_\mu P_i(l_\mu) \equiv 1$ . normalised probability.
- Soft-assignment allows you to consider all possibilities
- Let context act to find stable solution

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## Relaxation Labelling

- **Compatibility:**  
 Pixels  $i$  and  $j$ , labels  $\mu$  and  $\nu$ :  
 no effect  $c_{i,j}(\mu, \nu) = 0$  If not neighbours  
 support  $c_{i,j}(\mu, \mu) = \alpha$  Neighbours & same label  
 oppose  $c_{i,j}(\mu, \nu) = -\alpha$  if  $\mu \neq \nu$  Neighbours & different label
- **Contextual support for label  $\mu$  at pixel  $i$ :**

$$s_i(\mu) \propto \sum_{j \neq i} \sum_{\nu} c_{i,j}(\mu, \nu) P_j(\nu)$$

look at all other pixels (points to  $\sum_{j \neq i}$ )  
 degree of compatibility (points to  $c_{i,j}(\mu, \nu)$ )  
 all possible labels & how strong (points to  $P_j(\nu)$ )

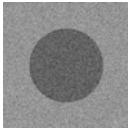
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## Relaxation Labelling:

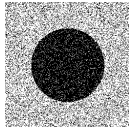
- Update soft labelling given context:  

$$P_i(\mu) \leftarrow A_i P_i(\mu) (1 + s_i(\mu))$$

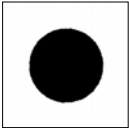
$$A_i$$
 chosen so sums to 1 at  $i$ .
- The more support, more likely the label
- Iterate



Noisy Image



Threshold labelling

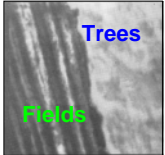


After iterating

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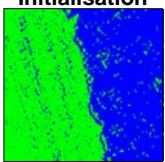
## Relaxation Labelling:

- Value of  $\alpha$  alters final result

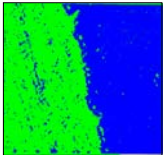


Trees  
Fields

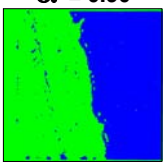
Initialisation



$\alpha = 0.75$



$\alpha = 0.90$



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## Segmentation as Optimisation

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## Segmentation as Optimisation

Image:  $\mathcal{I}$ , value at pixel  $i$ :  $\mathcal{I}(i)$   
 Label Image:  $L$ , label at pixel  $i$ :  $L(i)$   
 Label configuration in neighbourhood of  $i$ :  $l(i)$

- Maximise probability of labelling given image:  

$$P(L|\mathcal{I}) = \prod_i P(L(i)|\mathcal{I}(i)) P(L(i)|l(i))$$

label at i given value at i
label at i given labels in neighbourhood of i
- Re-write by taking logs, minimise cost function:  

$$C(L, \mathcal{I}) = \sum_i [-\log P(L(i)|\mathcal{I}(i)) - \log P(L(i)|l(i))]$$

label-data match
label consistency
- How to find the appropriate form for the two terms.
- How to find the optimum.

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## Segmentation as Optimisation

$P(L(i)|l(i))$  • Exact form depends on type of data  
**label consistency** • Histogram gives:  $p(\mathcal{I}(i))$   
 $P(L(i)|\mathcal{I}(i))$  • Model of histogram  $P(L(i)|\mathcal{I}(i))$   
**label-data match** (e.g., sum of gaussians, relaxation case)

Learning approach:

- Explicit training data (i.e., similar labelled images)
- Unsupervised, from image itself (e.g., histogram model):

Expectation/Maximization

- Given labels, construct model
- Given model, update labels
- Repeat

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## Segmentation as Optimisation

- General case:  
 Cost function:  $C(L, \mathcal{I}) = \text{label-data match term} + \text{label consistency}$
- High-dimensional search space, local minima
- Analogy to statistical mechanics
  - crystalline solid finding minimum energy state
  - stochastic optimisation
  - simulated annealing
- Search:
  - Downhill
  - Allow slight uphill

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## Segmentation as Optimisation

$\alpha = 0.90$

Original      Relaxation      Optimisation