Lectures 1 & 2: Basic Image Analysis

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Thursday 18th March

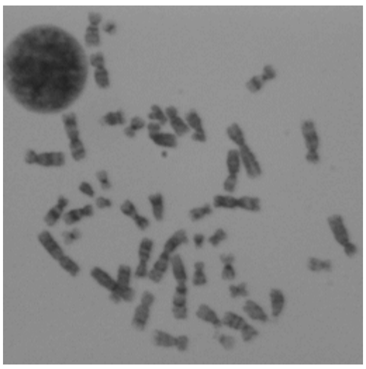
10:00am – 12:00am

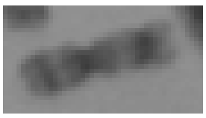
Basic Image Analysis

- Limited to simple 2D scenes
  - Adequately described as background and objects
- Good contrast between objects and background
  - staining or backlighting
- Constrained applications
  - microscopic materials analysis
  - biomedical microscopy
  - industrial inspection

Slide 2

Sample Problem:





- Stained preparation, light microscope
- Chromosomes, with bands
- Measure banding pattern

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Solving the Problem

Plan

Distinguish between objects & background

↓

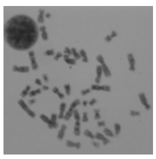
Locate each individual object

↓

Locate centerline

↓

Measure bands



Questions

What is an image?

What is background?

From not-background to distinct objects

Shape of an object?

Measurements on object

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Overview:

- Image Representation
  - What is an image?
- Grey-Level Processing
  - Improving the starting image
- Segmentation
  - Background pixels and object pixels
- Binary Image Processing
  - Improved background/object binary image
- Measurement
  - Object as connected region

Image Representation

Image Representation

- Isn’t it totally obvious? We all know what an image is!
- Various ways of representing an image, depending on the task in hand
  - Image function
  - Landscape
  - Array of pixels
  - Image histogram
  - In another space entirely!

Image Representation

Image

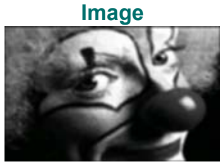


Image Plane

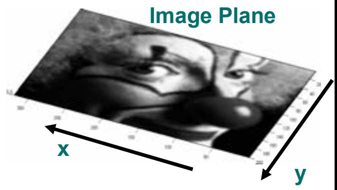


Image Function

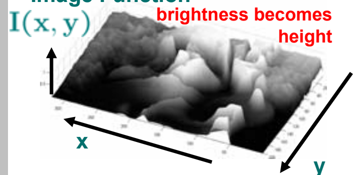


Image Landscape

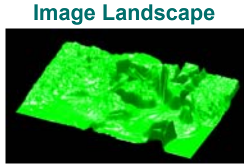
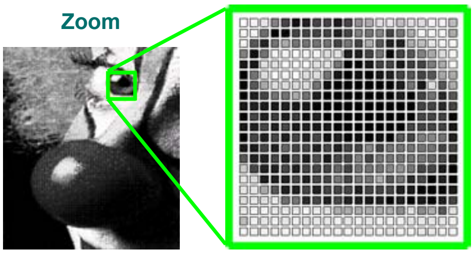


Image Representation

Zoom



Array of Pixels:

Values and spatial relationship

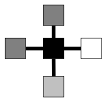


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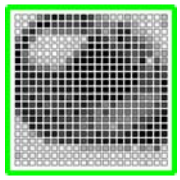
Image Representation



- Sort pixels by grayscale value/colour and stack them up

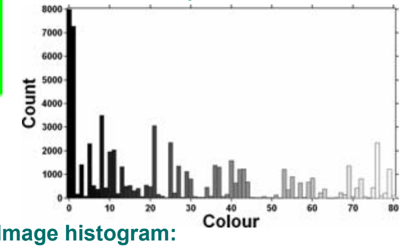


Image histogram:
Kept values but lost spatial information

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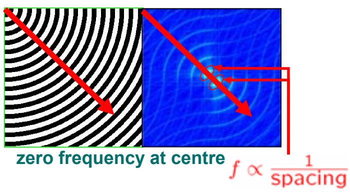
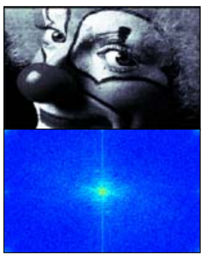
Image Representation

- **Fourier Analysis:**
  - any signal can be decomposed into a sum of sinusoids (FFT)
  - low frequencies, general shape, high frequencies details





NOTE:
zero frequency removed by subtracting mean value across image from image before doing FFT

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Image Representation

NOTE:
$e^{i\theta} \equiv \cos\theta + i\sin\theta$
$\Rightarrow$ $\mathcal{F}_I$ complex, $I(\underline{r})$ real
so $\mathcal{F}_I(-\underline{k}) \equiv \overline{\mathcal{F}_I}(\underline{k})$

- **Frequency Space:**
  - Integrate over the image, weighted by complex exponentials

$$\mathcal{F}_I(u,v) \propto \iint I(x,y)\exp(iux + ivy)dxdy$$

  - Compact vector form:

$$\mathcal{F}_I(\underline{k}) \propto \iint I(\underline{r})\exp(i\underline{k}\cdot\underline{r})d\underline{r}$$

  - Inverse:

$$I(\underline{r}) \propto \iint \mathcal{F}_I(\underline{k})\exp(-i\underline{k}\cdot\underline{r})d\underline{k}$$

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Grey-Level Processing

Grey-Level Processing

- **Restoration:**
  - What is noise, what is signal?
  - Remove blurring
- **Enhancement**
  - Emphasize required features (e.g., linear features)
  - Emphasize change (e.g., surveillance)

Grey-Level Processing: Overview

- **Point processes**
  - Transform global gray-level scale
- **Neighbourhood Processing**
  - Values and their context
- **Image Arithmetic**
  - Using a sequence/pair of images
- **Image Transforms**
  - Images in a different space (frequency space)

Grey-Level Processing: Point Processing

Grey-Level Processing: Point Processing

- Point = Pixel
- Transforms image based on single pixel value alone:

$$\underbrace{\tilde{I}(x,y)}_{\text{new pixel value}} = \underset{\text{function}}{f}\,(\underbrace{I(\overbrace{x,y}^{\text{position}})}_{\text{pixel value}})$$

- Various choices for monotonic function f(i)
  - Increase/decrease/stretch brightness and contrast
  - Gamma correction, power law : $f(i) = i^{\gamma}$
  - Histogram matching between images
  - Histogram equalization

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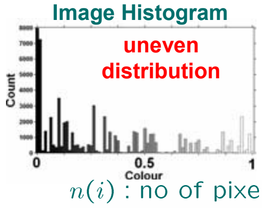
Point Processing: Histogram Equalisation

Image Histogram



- Re-assign colours, keep ordering (light to dark)
- Increase contrast

$n(i)$: no of pixels with colour i,

N : Total number

$$\boxed{\text{New Colour: } f(i) = \frac{1}{N}\sum_{j \leq i} n(j)}$$

$f(i) = 0.75$, 75% darker than this

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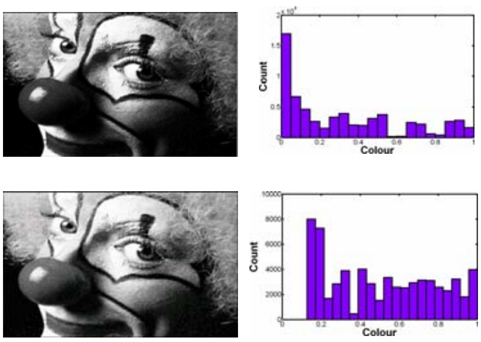
Point Processing: Histogram Equalisation



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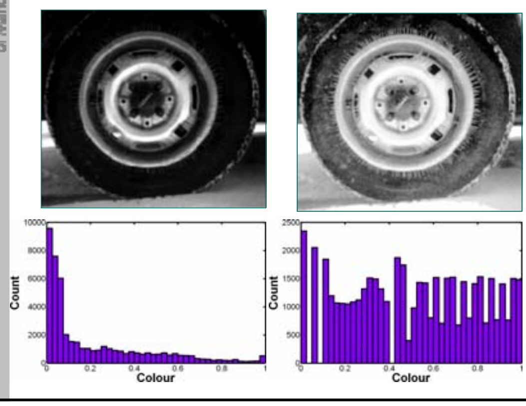
Point Processing: Histogram Equalisation



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Grey-Level Processing: Neighbourhood Processing

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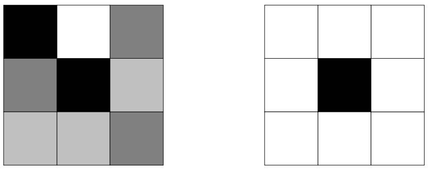
Neighbourhood Processing

single black pixel



Noisy dark area | Just noise

- Consider a single pixel value in context of neighbours
- Neighbourhood (e.g. 3 x 3), structuring element (SE)
- Two methods:
  - Convolution
  - Rank Filtering

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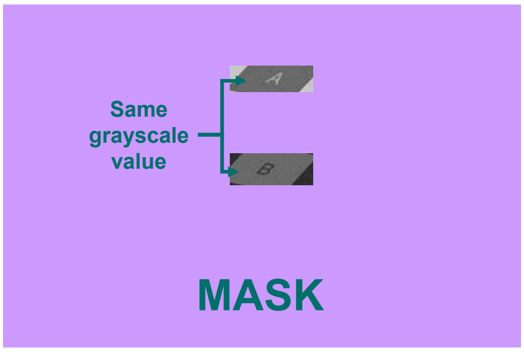
Aside: Context in Human Vision



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Convolution: 1D Example

NOTE:
0 black to
1 white
0 to 255 8-bit
images

$g(a)$	1	2	1							
$I(x)$	1	0	0.5	1	1	0.5	0.5	0.5	1	1
$\tilde{I}(x)$	1	0.37	0.5	0.9	0.9	0.6	0.5	0.6	0.9	1

- Weighted sum of neighbours

$$\tilde{I}(x) = \frac{\sum_a g(a) I(x+a)}{\sum_b g(b)}$$

Normalize the weights

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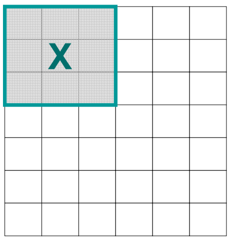
Convolution: 2D

g(-1,1)	g(0,1)	g(1,1)
g(-1,0)	g(0,0)	g(1,0)
g(-1,-1)	g(0,-1)	g(1,-1)

g(a,b)



$\mathrm{I}(x,y)$

$$\tilde{\mathrm{I}}(x,y) = \frac{\sum_a \sum_b g(a,b)\mathrm{I}(x+a,y+b)}{\sum_c \sum_e g(c,e)}$$

Normalize the weights

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Convolution: 2D

- Asterisk notation:

$$\tilde{\mathrm{I}} = g * \mathrm{I}$$

- Discrete form:

$$\tilde{\mathrm{I}}(x,y) = \frac{\sum_a \sum_b g(a,b)\mathrm{I}(x+a,y+b)}{\sum_c \sum_e g(c,e)}$$

- Integral form: $\tilde{\mathrm{I}}(x,y) = \frac{\iint g(a,b)\mathrm{I}(x+a,y+b)dadb}{\iint g(c,e)dcde}$

- Integral form (vector notation)

$$\underline{r} = (x,y), \quad \tilde{\mathrm{I}}(\underline{r}) = \frac{\iint g(\underline{z})\mathrm{I}(\underline{r}+\underline{z})d\underline{z}}{\iint g(\underline{y})d\underline{y}}$$

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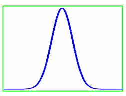
Convolution: Common Kernels

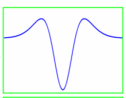
- Gaussian: $g(x,y) = A\exp(-(x^2+y^2)/2\sigma^2)$ σ width
  - Smoothing kernel
  - Any unimodal kernel smoothes the image



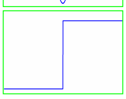
- Difference of Gaussian (DoG)

$$g(x,y) = A\exp(-(x^2+y^2)/2\sigma^2) - B\exp(-(x^2+y^2)/2\alpha^2)$$

- Laplacian (or Laplacian of Gaussian)
  - similar shape to DoG, second-derivative filter



- First-derivative edge filters
  - ridges at edge positions



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Convolution Theorem

NOTE:
$e^{i\theta} \equiv \cos\theta + i\sin\theta$
$\Rightarrow$ $\mathcal{F}_{\mathrm{I}}$ complex, $\mathrm{I}(\underline{r})$ real
so $\mathcal{F}_{\mathrm{I}}(-\underline{k}) \equiv \overline{\mathcal{F}}_{\mathrm{I}}(\underline{k})$

- Frequency space (see Image Representation) :

$$\mathcal{F}_I(\underline{k}) \propto \iint I(\underline{r})\exp(i\underline{k}\cdot\underline{r})d\underline{r}$$

- Look at it in frequency space or real space:
  - convolution in real space $\Leftrightarrow$ multiplication in frequency space

$$g * \mathrm{I} \iff \mathcal{F}_g \times \mathcal{F}_{\mathrm{I}}, \quad g * \mathrm{I} \equiv \mathcal{F}^{-1}(\mathcal{F}_g \times \mathcal{F}_{\mathrm{I}})$$

  - convolution in frequency space $\Leftrightarrow$ multiplication in real space

$$\mathcal{F}_g * \mathcal{F}_{\mathrm{I}} \iff g \times \mathrm{I}, \quad \mathcal{F}_g * \mathcal{F}_{\mathrm{I}} \equiv \mathcal{F}(g \times \mathrm{I})$$

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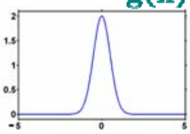
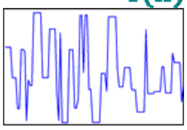
Convolution Theorem: Gaussian

Real space

$g(x)$

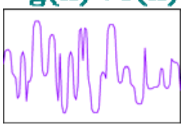


$f(x)$



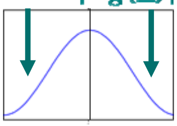
0 100

$g(x) * f(x)$



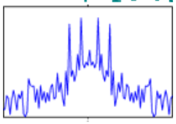
Frequency space

$|\mathcal{F}_g(k)|$

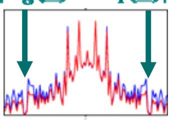


0

$|\mathcal{F}_I(k)|$



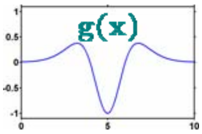
$|\mathcal{F}_g(k) \times \mathcal{F}_I(k)|$



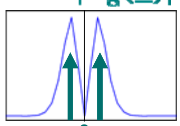
Convolution Theorem: Difference of Gaussians

$$g(x,y) = A\exp(-(x^2+y^2)/2\sigma^2) - B\exp(-(x^2+y^2)/2\alpha^2)$$

$g(x)$



$|\mathcal{F}_g(k)|$



0

- band-pass filter, enhances edges
- Laplacian and LoG similar

signal at edges

Convolution Theorem: Laplacian of Gaussian & Difference of Gaussians

Gaussian and FT of Gaussian

$$\boxed{g(x) \propto e^{-\beta x^2}, \mathcal{F}_g(k) \propto e^{-\alpha k^2}}$$

Convolution Theorem

$$\boxed{g * I \equiv \mathcal{F}^{-1}\left(\mathcal{F}_g \times \mathcal{F}_I\right)}$$

Laplacian of gaussian:

$$\frac{\partial^2}{\partial x^2}\left(\int e^{-ikx}e^{-\alpha k^2}\mathcal{F}_I(k)dk\right)$$

Laplacian Inverse FT Gaussian FT of Image

- Do the derivative:

$$\int \boxed{-k^2}e^{-ikx}\boxed{e^{-\alpha k^2}}\mathcal{F}_I(k)dk \rightarrow \frac{d}{d\alpha}\overbrace{\int e^{-ikx}e^{-\alpha k^2}\mathcal{F}_I(k)dk}^{\text{Convolution with Gaussian, parameter } \alpha}$$

- LoG: difference of infinitesimally-separated gaussians
- DoG: difference of finitely-separated gaussians

Neighbourhood Processing: Rank Filtering

Neighbourhood Processing: Rank Filtering

- Output is rank function of neighbourhood:
  - median (smoothes and preserves edges)
  - max and/or min (mathematical morphology)
  - rank number (seven of nine)
- Harder to analyse than convolution


Noisy Image


3x3 mean


3x3 median

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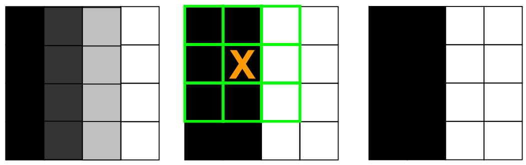
Rank Filtering & Edges: Example

- Mean:
- 2/3 ■ + 1/3 □ = ■
- 1/3 ■ + 2/3 □ = ■
- Median:
- 6 ■ & 3 □ => ■
- 6 □ & 3 ■ => □



3x3 SE

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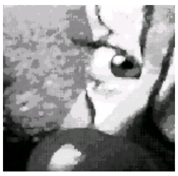
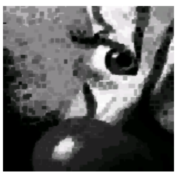
Neighbourhood Processing: Rank Filtering

- Rank Number
  - 3 x 3 structure element


Original


maximum


7^{th} of nine

blocky, impressionistic effect

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Grey-Level Processing: Image Arithmetic

Image Arithmetic: Addition

- Take average over images in sequence
- Reduces noise


Original

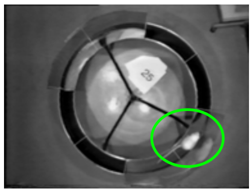

Addition


Noisy 1


Noisy 2

Image Arithmetic: Subtraction

- Take difference:
  - Negative values? Shift and scale to get back to [0:255]
  - Or take absolute difference
- Static background, detects change
- Object, shadows & reflections in real-world scenes



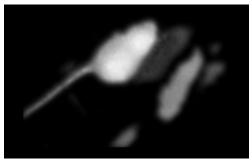
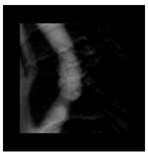


Image Arithmetic: Subtraction

- Digital subtraction angiography (DSA)
- Pre-study radiograph
- Contrast agent injection
- Post-contrast radiograph
- Difference



Introduction to Segmentation

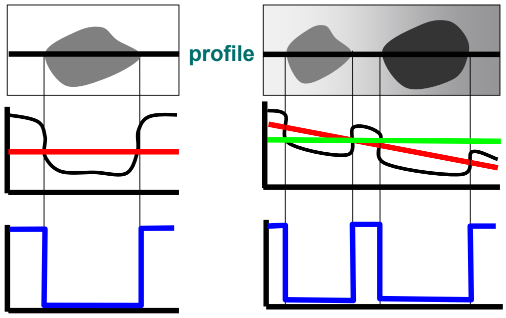
Segmentation:

Task: label each pixel as either object or background

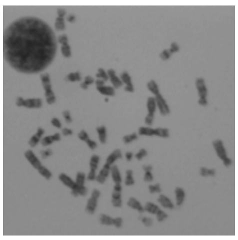
- Grayscale image ⟶ binary label image
- Thresholding
  - simple, high-contrast images
- Adaptive thresholding
  - simple images with shaded background
- Advanced Segmentation
  - open research problem

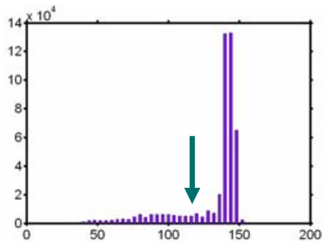
Segmentation: Thresholding

- Thresholding
- Adaptive Thresholding



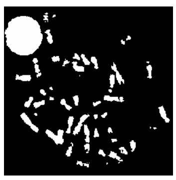
Segmentation: Thresholding, Histogram



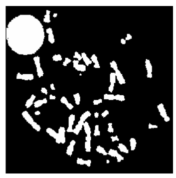


Segmentation: Thresholding

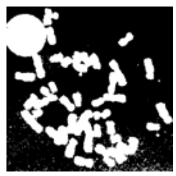
- Varying the Threshold



Threshold 100



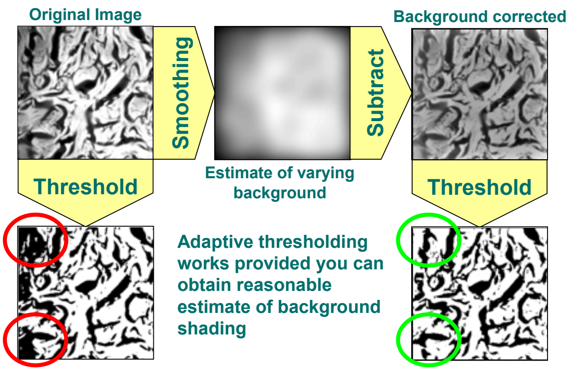
Threshold 110



Threshold 140

- Need to choose threshold with care,
- How to improve the binary image

Segmentation: Adaptive Thresholding



Adaptive thresholding works provided you can obtain reasonable estimate of background shading

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Binary Processing

Binary Processing

Aim: Improved binary image

- Restoration or enhancement
- Neighbourhood Processing:
  - binary morphology (erosion & dilation)
  - skeletonization
- Image Logic:
  - combining binary images for more complicated processing

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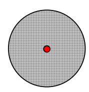
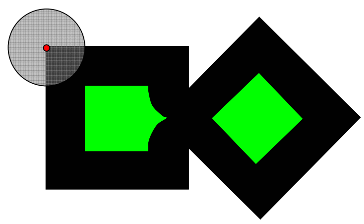
Binary Morphology: Erosion

- Structure element (example, centre marked): 
- Binary object:
- Sweep SE along boundary, and delete region covered



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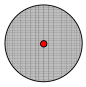
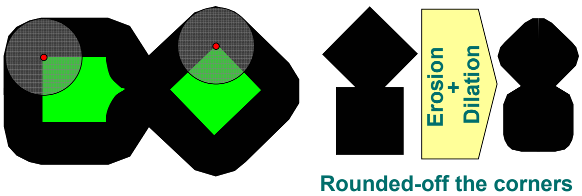
Binary Morphology: Dilation

- Structure element (centre marked):



- Binary object:
- Reverse of erosion
- Sweep SE along boundary, and add region covered



Rounded-off the corners

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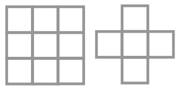
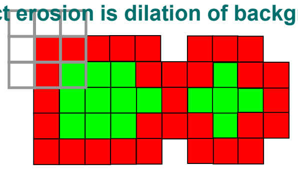
Binary Morphology: Dilation, Implementation via Neighbourhood Processing



- Pixellated structuring element
- Pixellated image object
- Scan SE over image, and add pixel at defined centre if any object pixel lies within SE
- Object erosion is dilation of background, so similar



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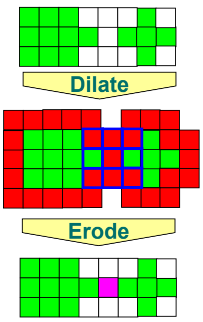
Binary Morphology: Closing & Opening

- Closing: reconnection



- Opening: disconnection

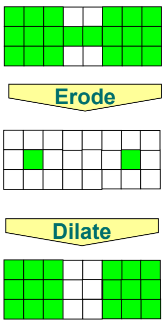


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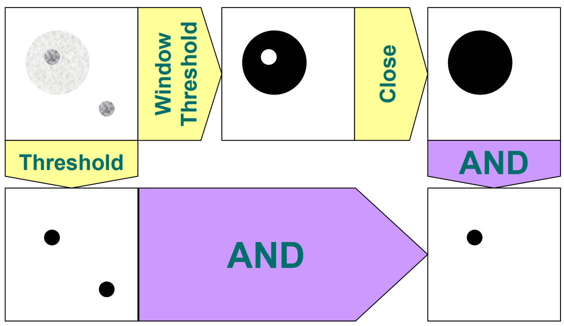
Image Logic:

Want dark object within grey object



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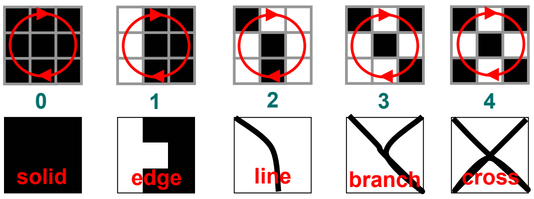
Binary Morphology: Skeletonisation

- Erosion that preserves connections
- Rutovitz Crossing Number: (3x3 SE)
  - loop and half the number of times value changes



- Remove centre pixel if 1: nibble at edge, but leave crossings
- Repeat until no further change

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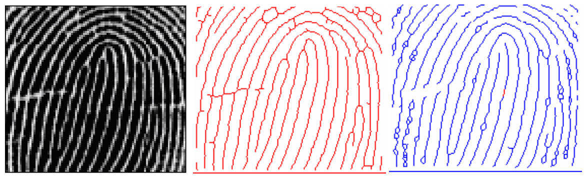
Binary Morphology: Skeletonisation



fingerprint valleys ridges

Feng Zhao and Xiaoou Tang

PREPROCESSING FOR SKELETON-BASED FINGERPRINT MINUTIAE EXTRACTION

CISST'02 International Conference

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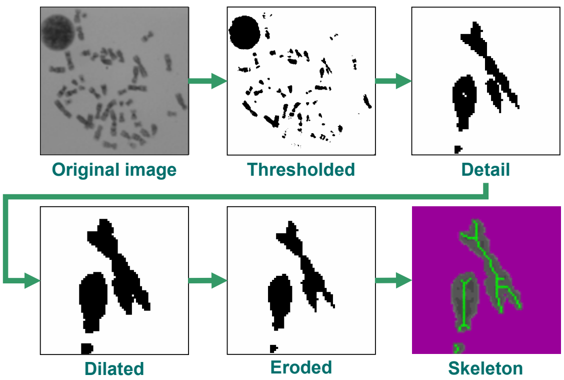
Chromosome Results:



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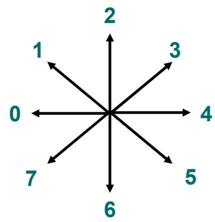
Measurement

Simple Measurements on Objects

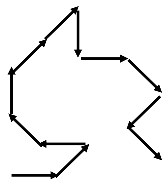
- Extracted objects as above
- Representing Objects:
  - Boundary representation
  - Area representation
- Simple geometric measurements
  - Area
  - Perimeter
  - Circularity

Representing Objects: Boundary

- Boundary Representation: chain code



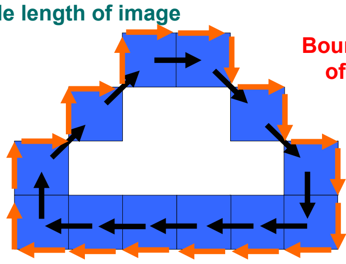
Pick a set of directions



chain code:
4 3 0 1 2 3 3 6 4 5 7 5

Representing Objects: Boundary

- Positions of boundary pixels: 2N times (one from L)
- L: side length of image

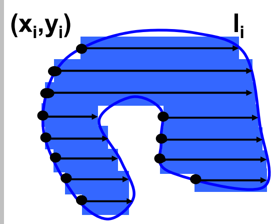


Boundary pixels of an object

- Chain code: N times (one of eight)
- OR: ~1.5 N times (one of four)

Representing Objects: Area

- Area Representation: Chord List



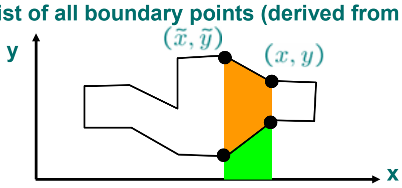
chord (x_i ,y_i ,l_i):
start position and length

Chord list represents the shape of the pixelated object

Much more efficient representation of data compared to storing position of every pixel within the region!

Measurement: Area

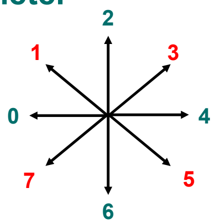
- List of all boundary points (derived from chord list)



- Trapezoidal rule $\text{Area} = \frac{(y+\tilde{y})(x-\tilde{x})}{2}$
- Take difference to find area of strip of shape

Measurement: Perimeter

- 8-piece Chain Code:
- Diagonals are longer!



$$P = N_{even} + \sqrt{2}N_{odd}$$

- 4-piece chain code: P = N, all equal length
- Circularity: $C = \frac{4\pi \text{Area}}{P^2},$
- C=1 for circle, C<1 for anything else

Summary

Basic Image Analysis:

- Mostly straightforward and fairly intuitive
- Can give good results on suitable images
- Have to grasp basics before can move on to more sophisticated methods