Fluid Dynamics, Vol. 33, No. 1, 1998

MATHEMATICAL SIMULATION OF GAS CLOUD MOTION FOLLOWING THE ATMOSPHERIC EXPLOSION OF A METEOROID

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The gasdynamic consequences of the explosion of a meteoroid in a dense atmosphere are studied. The gas motion is simulated on the basis of a high-accuracy numerical method using adaptive movable networks. Some effects which could not be reproduced using coarser networks are revealed.

In the problem of gas cloud motion, the domain of the phenomenon under study may change considerably in size and spatial position; therefore, the computational domain must also change in the course of the calculations. Moreover, the problem is complicated by the presence of relatively large gradients caused by shock waves and contact discontinuities.

The flight of cosmic bodies such as asteroids, cometary nuclei and fragments, and meteoroids in planetary atmospheres is accompanied by high aerodynamic and thermal loads, the action of inertia forces caused by the deceleration of the body, and intense ablation. Various scenarios of the interaction of the body with an atmosphere are possible depending on the body parameters, such as its size R_0 and density ρ_0, the rupture stress σ^*, the heat of ablation, the specific energy of sublimation Q, the entry velocity V_∞ the scale length $H=D(h)/\sin w$ characterizing the variation of the atmospheric density along the body path (D is the altitude scale with respect to density and w is the angle of inclination of the entry path to the horizon), etc. Small bodies with characteristic dimensions much less than the atmospheric nonuniformity scale are totally evaporated in the upper atmosphere due to the intense radiative and convective heat fluxes. A whole series of papers (see, for example, [1—5]) has been devoted to estimates and approximate approaches to the problem of the breakup of large cosmic bodies.

A hydrodynamic approach to the problem, based on the assumption that the mass of fragmentation products, as it is deformed under the action of the distributed aerodynamic load behaves like an incompressible fluid, was suggested in [3]. On the basis of simple estimates, an analytical model of meteoroid deceleration in a planetary atmosphere with allowance for the variation of the effective cross-section of the body was developed. By combining the numerical and analytical approaches it is possible to obtain the flow pattern in more detail.

1. MODEL OF METEOROID EXPLOSION

The altitude in the atmosphere of a planet at which the deformation of fragmentation products is of the same order as their characteristic dimension can be estimated by evaluating the individual terms in the equations governing thedeformation of a quasi-liquid volume of the debris in a center-of-mass-fitted reference frame

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho_0}\nabla p$$

In the initial deformation stage, we can neglect convection and evaluate the pressure gradient on the assumption that on the body scalelength the pressure varies from the stagnation pressure P_0 at the stagnation point to a value $p \ll P_0$ on the lateral surface

$$\frac{\partial u}{\partial t} \sim \frac{1}{\rho_0}\frac{P_0}{R_0} \sim \frac{\rho(h)V_\infty^2}{\rho_0 R_0} \tag{1.1}$$

where $\rho(h)$ is the atmospheric density at the altitude h and u is the radial velocity.

Moscow. Translated from Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Zhidkosti i Gaza, No. 1, pp. 29-37, January-February, 1998. Original article submitted February 18, 1996.

 0015-4628/98/3301-0024$20.00

Integrating (1.1) for an exponential atmosphere, for the spreading velocity of the debris we obtain the estimate

$$u(h) \sim \frac{\rho(h)}{\rho_0} \frac{H}{R_0} V_\infty$$

Integrating with respect to time under the same conditions gives us an estimate for the deformation scale of the fragmentation products as a function of the altitude in the planetary atmosphere

$$\delta R(h) \sim \frac{\rho(h)}{\rho_0} \frac{H^2}{R_0}$$

Extrapolating the relations thus obtained to the altitude h_* at which the scale of the deformation caused by the distributed gasdynamic load is of the order of the characteristic size of the object, $\delta R(h) \sim R_0$, we obtain an expression for the atmospheric density at the altitude h_* where the body breaks up totally

$$\frac{\rho(h_*)}{\rho_0} = \left[\frac{R_0}{H}\right]^2$$

For large-scale deformation, an estimate based on the equality of the inertia forces and the gas pressure on the body surface [4] is usually employed

$$\frac{u^2}{R_0} \sim \frac{1}{\rho_0} \frac{P_0(h)}{R_0}$$

This leads to the following simple approximate formula for the expansion velocity

$$u(h) = \left(\frac{\rho(h)}{\rho_0}\right)^{1/2} V_\infty$$

In the process of breakup, the gasdynamic action of the incident flow is insufficient for disintegration to be accompanied by any significant entrainment of debris or separation of large fragments even if the stagnation pressure is transmitted through fractures from the shock layer into the bulk of the body. The breakup process is progressive and leads to the formation of a large number of pieces within the initial meteoroid volume. A disintegrated fragment continues to move as a compact formation without any appreciable increase in the effective cross-sectional area. In accordance with the data of [6], the loss of body mass due to ablation does not exceed 1% up to the moment of total disintegration.

The results of a numerical investigation based on a hydrodynamic approximation [7] considerably refined the dynamic pattern of body deformation under the action of the incident flow, especially as regards the estimation of the convective instability effect. The calculation of the motion of an icy body 200 m in diameter in the Earth's atmosphere within the framework of the hydrodynamic approximation [4] (the equation of state for water was used) showed that convective instability on the windward side of the fragment strongly influences the liquid-body dynamics. Simultaneously with the flattening of the liquid volume in the vicinity of the stagnation point, perturbations of the contact discontinuity develop. A depression is formed at the spreading point (break-through takes place in the near-axis region of the liquid body) so that the continuity of the volume is violated. The area of the liquid-gas interface increases many times, while the maximum body radius increases by about 1.4 times. A similar interaction pattern was obtained in [6] where the deceleration of a fragment of the comet Shoemaker-Levy 9 was simulated numerically.

The deformation and loss of continuity of the quasi-liquid debris body is accompanied by the dispersion of particles of condensed phase in the high-temperature gas consisting of the sublimation products and the gas of the incident flow. As a result, the area of the high-temperature gas/condensed phase interface sharply increases. All this leads to a fairly rapid transition of the meteoroid material from the condensed to the gaseous state. This process has been called an "explosion" [8, 9]. The interpretation of the disintegration of a fractured cosmic body in a dense atmosphere as an explosion has resulted in a formulation of the problem based on the gasdynamic equations; this formulation can be characterized as "explosion in flight" [8].

In [8, 9] two control parameters, namely, the ratio of the kinetic energy of the body to its internal energy and the explosion altitude (or the body-to-atmosphere density ratio) were specified. In this study, we propose a formulation of the problem with a single indeterminate parameter, namely, the explosion altitude. Calculations show that if the error in evaluating the explosion altitude is less than the altitude scale (the atmospheric nonuniformity scale), it does not lead to a significant error in determining the deceleration height.

The internal energy of the gas incorporates the chemical binding energy, while the equation of state takes dissociation and ionization into account. The heating of the meteoroid material by intense thermal fluxes raises the sublimate pressure which can locally (in the vicinity of the phase interface) exceed the stagnation pressure, the main dynamic parameter of the meteoroid disintegration process. We will assume that at the moment of explosion, the meteoroid material is localized within a domain whose dimensions are of the same order as the initial dimensions of the body and then expands into the wake region and in the radial direction under the action of the stagnation pressure. This treatment of the lateral expansion is widely used in analytical models [4, 5].

We propose the following formulation of the problem in the gas approximation. We place a spherical volume in a stratified atmosphere at a conditional explosion altitude h (here, explosion is to be understood in the sense of [8, 9]). The volume contains a homogeneous gaseous medium with the following parameters: (1) the mass of the gas is assumed to be equal to that of the original cosmic body, (2) the gas velocity is equal to the entry velocity, (3) the density corresponds to the cosmic body density, and (4) the static pressure is equal to the stagnation pressure at the given altitude

$$P_0 = \rho(h) V_\infty^2$$

At the stagnation point, the contact discontinuity is in equilibrium. On the other hand, the breakdown of the discontinuity in the lateral direction and in the direction opposite to the body motion constitutes gas expansion into a vacuum. A characteristic feature of this formulation is that it has only one free parameter, namely, the explosion altitude, since within the framework of this formulation the internal-to-kinetic energy ratio of the meteoroid material is uniquely determined by the atmosphere/gaseous body density ratio at the explosion altitude

$$\frac{e}{K} = \frac{2}{\gamma - 1} \frac{\rho(h)}{\rho_0}$$

The explosion altitude can be estimated from the approximate solution [3]. It should be noted that the error in determining the explosion altitude within the limits of the altitude scale only slightly affects the final height of cloud deceleration. Thus, the calculations for a kilometer-sized fragment of the comet Shoemaker-Levy 9 performed for explosion altitudes corresponding to $\rho(h)/\rho_0 = 1.88 \cdot 10^{-3}$ and $6.5 \cdot 10^{-4}$ gave similar values for the depth of penetration into the Jovian atmosphere (the discrepancy in penetration depths was considerably smaller than the difference in explosion altitudes).

Let us now consider some results of the numerical simulation of the gasdynamic consequences of an explosion.

We used a second-order TVD-type difference scheme [10] for the numerical solution of the Euler equations. The movable computational grid was adapted to the solution. The grid was constructed on the basis of the solution of the variational problem [11, 12]. As a result, in the physical domain the computational grid was adapted to high gradients in the required solution, was close to an orthogonal network and fairly convex.

2. PROBLEM OF GAS CLOUD MOTION FOLLOWING A METEOROID EXPLOSION

Let a homogeneous gaseous medium at rest fill the entire space and be characterized at the initial moment by the pressure $p = P_a$, the density $\rho = \rho_a$, and the temperature $T = T_a$. The gas in the atmosphere is assumed to be perfect, with the specific heat ratio $\gamma = 1.4$, inviscid, and non-heat-conducting. At the initial moment, a spherical gaseous cloud of radius R_0 possessing the velocity V_∞ is introduced into the atmosphere. It is assumed that at the initial moment the pressure inside the cloud is equal to the pressure on the stagnation line in the flow past the solid meteoroid ($P_0 = \rho_a V_\infty^2$). The temperature and density of the gas cloud are initially equal to $T = T_0$ and $\rho = \rho_0$, respectively. We will assume that the cloud consists of a perfect ($\gamma = 1.4$), inviscid, and non-heat-conducting gas. In this formulation, the flow accompanying the flight of the gas cloud essentially depends on the following dimensionless parameters: the body density $G = \rho_0/\rho_a$, the pressure in the gas cloud $P = P_0/P_a$, and the Mach number $M = V_\infty/a_a$. We will specify $G = 10^2$, $P = 1.26 \cdot 10^3$, and $M = 30$. From the given initial values it follows that the internal energy of the gas inside the cloud is much smaller than the kinetic energy: $E_0/K_0 = [0.5\rho_0 V_\infty^2]^{-1}[P_0/(\gamma - 1)] = 0.05$. An analogous formulation was considered in [8], but with another value of E_0/K_0; moreover, in that study another numerical approach (Godunov method) based on a uniform grid was applied.

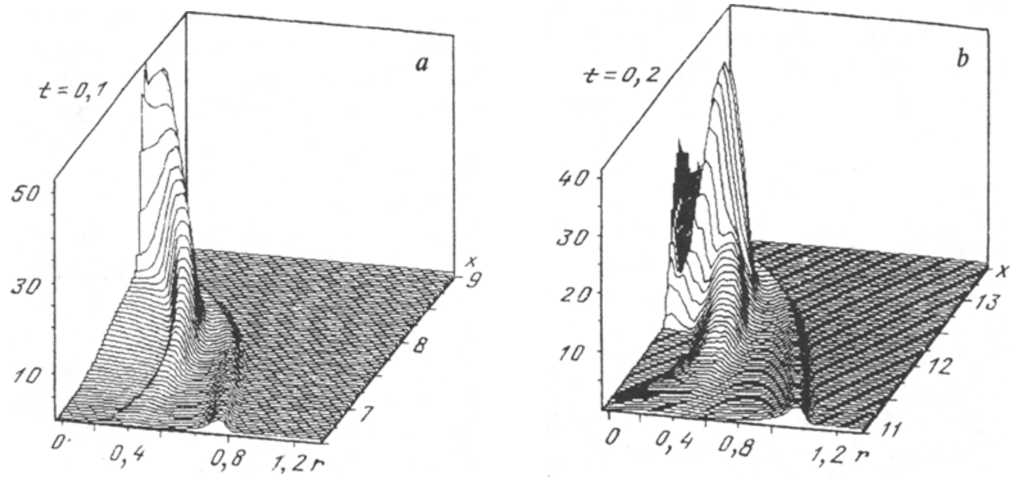

Fig. 1. Dynamic pattern of the forcing-through of a gas cloud. Density of the meteoroid material at the moments: *a*, $t=0.1$ and *b*, $t=0.2$.

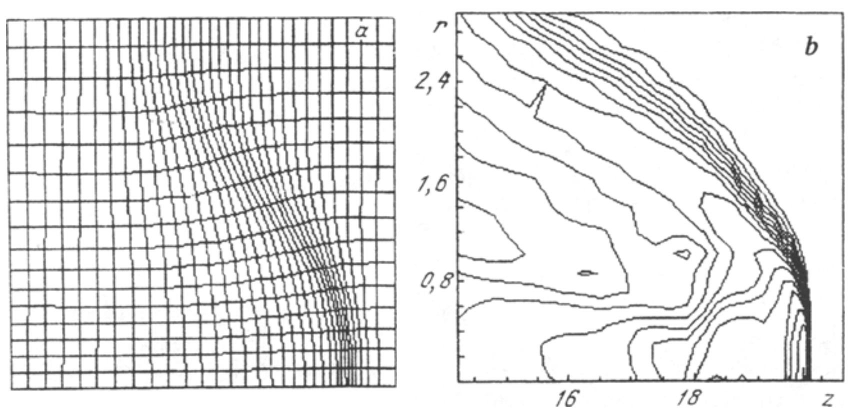

Fig. 2. Adaptive network generated at the moment $t=0.3$ (*a*) and the corresponding isolines (*b*).

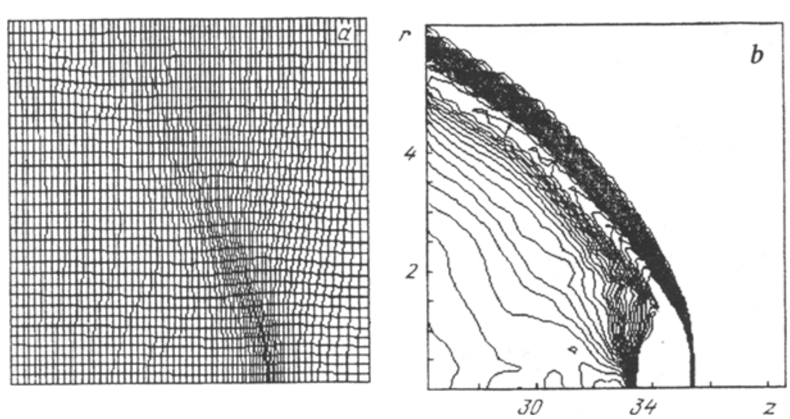

Fig. 3. Adaptive network generated at the moment $t=0.7$ (*a*) and the corresponding isolines (*b*).

The calculations were performed using difference networks with 32x16 and 64x32 cells; in these products the first and second cofactors relate to the x and y directions. Prior to adaptation, the network was rectilinear and uniform. The adaptation of the network began at the moment $t=0.05$; the calculations were carried out up to the moment $t=1.1$ (time t is divided by R_0/V_∞). The quantity $\ln p$ was used as the function with respect to which the network was adapted to the solution.

For a short time, most of the cloud material is concentrated in a cup-shaped layer with a radius of about $2.5R_0$ and a thickness of about $0.6R_0$. The edges of the cup are directed counter to the motion. The gas cloud owes its shape to the fact that the leading front of the cloud undergoes deceleration, while most of it continues to move at a velocity greater than that of the front. After the formation of the cup-shaped layer, the following process takes place. Under the action of the peak pressure at the nose, the cloud material is "forced through" on the axis of symmetry. This process starts at the moment $t=0.1$ and continues for a time interval of about 0.1. The dynamic pattern of this process is illustrated in Fig. 1. In this figure, as well as in Figs. 2 and 3, the results are presented in the cylindrical reference frame r, z. The r coordinate is measured from the axis of symmetry and z from the center of the spherical volume at the moment of explosion. The gas density divided by ρ_a is laid off along the vertical axis in Fig. 1. At the moment $t=0.6$ a rarefaction zone is formed immediately behind the cloud, into which the atmospheric gas starts to stream. In the near-axis region of the rear part of the cloud, the gas flowing into the rarefaction zone undergoes deceleration, which subsequently results in the formation of a local shock. The pattern of the hydrodynamic flows thus formed is described in more detail in [13–15]. In this study, the motion of the gas cloud is investigated using movable adaptive grids [12]. In Figs. 2 and 3 we present the isolines of $\ln p$ obtained at various moments and the adaptive networks generated using $\ln p$ as a weighting function (for smoothing the pressure gradients). Clearly, the network traces the formation of a steep pressure gradient at the nose of the gas cloud. It should be noted that the "forcing through" of the meteoroid cloud by the incident flow in the peak pressure region is a "fine" effect not revealed by calculations made using coarser networks. In what follows, we will consider another source of large-scale disturbances of the atmosphere initiated by meteoroid entry.

3. SIMULATION OF THE CONSEQUENCES OF THE ATMOSPHERIC ENTRY OF A METEOROID ALONG AN INCLINED PATH

When the angle which the meteoroid entry path makes with the vertical is fairly large, part of the bow shock surface has a negative component of the normal relative to the gravity vector. The intensity of this part of the shock increases with time so that it entrains atmospheric gas which is ejected to higher altitudes. A similar situation arose during the collision of the comet Shoemaker-Levy 9 with Jupiter; in that case the entry angle was 45° and the comet's velocity was 60 km/s. The results of a numerical simulation suggest the following scenario for the interaction between a fragment and the Jovian atmosphere. The part of the bow shock propagating in the direction of exponential decrease in atmospheric density accelerates thanks to the concentration of the wave impulse over a mass decreasing without bound.

Thus, the atmosphere is "broken through", i.e., the shock wave moves off to infinity in a finite time. A jet flow is initiated behind the shock front and in this flow the gas velocity and temperature increase with altitude. In the calculations, the time it took for the shock to arrive at the limb ($h=700$ km) was 25 s, which is in agreement with the estimates and far exceeds the energy release time (the limb is the boundary between the parts of Jupiter visible and invisible from the Earth). An exponential increase in the shock speed D is accompanied by an even faster increase in the temperature (proportional to D^2); for this reason, starting from a certain altitude, intense ionization occurs behind the shock front, so that when the shock is at an altitude observable from the Earth, the gas at a certain distance downstream of the front is fully ionized.

In many foreign publications [7, 16] atmospheric break-through is attributed to the formation of a shock traveling through the hot wake flow. In our opinion, break-through can be caused by the upper part of the bow shock.

Let us consider the propagation of a shock initiated by a cometary fragment moving in a planetary atmosphere. In an exponential atmosphere the fragment velocity varies with altitude as follows [3, 7]:

$$V = V_\infty \exp\left(-\frac{\pi}{2} C_x M(h) m^{-1}\right)$$

where m is the fragment mass and $M(h)$ is the mass of atmospheric gas within the volume formed by the effective radius of the meteoroid as it travels down to the altitude h. In an atmosphere with an exponential density distribution with respect to altitude, we have

$$M(h) = \rho(h) H \pi R^2 / \cos w$$

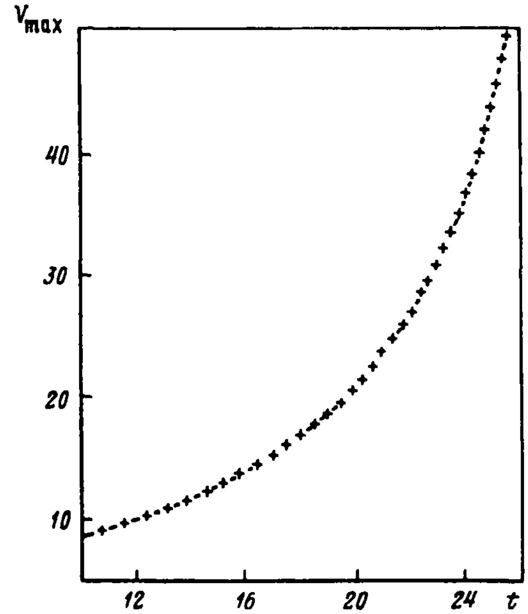

Fig. 4. Time dependence of the velocity at the peak temperature points behind the shock front.

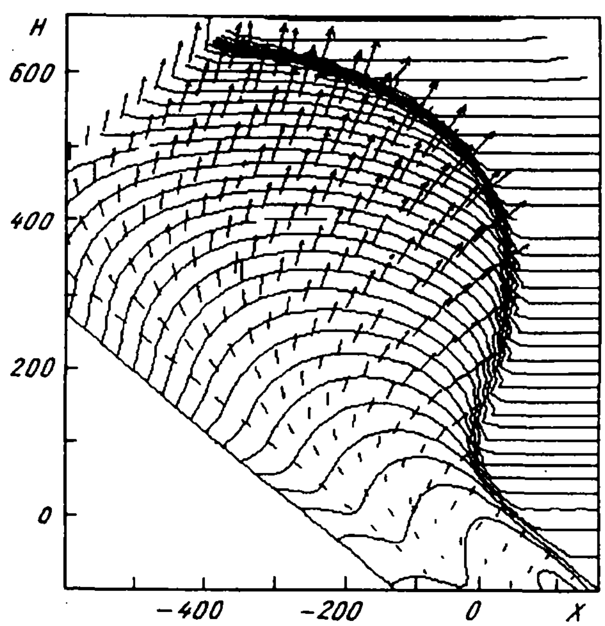

Fig. 5. Isolines of $\ln p$. The interval between isolines corresponds to a change in pressure by e times.

The problem was solved in the plane of symmetry defined by the gravity vector and the axis coinciding with the direction of meteoroid entry. The equation of state for hydrogen was used with account for the effect of dissociation and ionization reactions in the local thermodynamic equilibrium approximation. The shock wave was initiated by a cylindrical explosion. The inclination of the cylinder corresponded to the entry angle of fragments of the comet Shoemaker-Levy 9. The distribution of the gas energy behind the shock was calculated in accordance with [7]

$$e(h) = \pi C_x \rho(h)(V_\infty R)^2 \exp\left(-\frac{3}{4} C_x \frac{H\rho(h)}{R_0 \rho_0 \cos w}\right)(2\cos w)^{-1}$$

with $C_x = 1$, $R_0 = 1$ km, $V_\infty = 65$ km/s, and $\rho_0 = 1$ g/cm^3, which corresponds to the flow past a solid sphere with a radius of 1 km.

At a time $t = 15$ s from the generation of the shock, a bulge on the explosive cylinder (disturbed region) can be observed in the vicinity of the minimum of the altitude scale, which corresponds to an altitude of about 50 km (the tropopause). At the bulge site maxima of the gas velocity and temperature appear.

The maximum velocity of the shock front rapidly increases. The time dependence of the velocity at peak temperature points behind the shock front is exponential (Fig. 4). At $t = 28$ s, the shock reaches an altitude $h = 700$ km which can be directly observed from Earth.

The gas velocity at the point of maximum shock intensity is directed at an angle of 30° to the local vertical. At the moment of shock arrival at the limb, the velocity reaches 60 km/s. The distance between neighboring isolines in Fig. 5 in the undisturbed atmosphere corresponds to the local altitude scale with respect to pressure. The shock is most strongly accelerated (with subsequent arrival at the limb) on the near-tropopause section of the trajectory; however, the energy released near this altitude constitutes only a small fraction of the total energy of the body. On the lower part of the trajectory, the shock evolves slowly imparting relatively small vertical velocities to the atmospheric gas.

Summary. The following qualitative differences between the results obtained in this study and the solution of the point explosion approximation should be noted. Firstly, the gas is ejected not in the vertical direction but at an angle to the horizon, which affects the final size and mass of the gas cloud. Secondly, the section of the trajectory on which the shock that leads to atmospheric break-through is formed does not coincide with the point of maximum energy release but lies somewhat higher, near the altitude corresponding to a pressure of 1 bar. This may result in the nature of the dependence of the shock arrival to the limb on the energy release differing from that in the Kompaneets solution for a point explosion [17].

The study was carried out with the support of the Russian Foundation for Fundamental Research (project No. 96-01-00448).

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