Chapter 1

A vector space semantics for dimensional adjectives

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Abstract. The paper presents a compositional semantics for comparatives of dimensional adjectives within the framework of vector space semantics, including measure phrases and comparison of deviation.

1 Introduction

The paper[1] has two main goals. The first is to show that vector space semantics (VSS), which was developed by [Zwarts 1995] and [Zwarts et. al 1997] to account for phenomena in the spatial domain, is a versatile enough framework to also be applicable to other areas. The second goal is to develop a compositional semantics for constructions involving dimensional adjectives (DAs) in predicative position of the kind exemplified in (1).

(1) a. Sam is taller than Jo. b. The board is long.

In semantic research it is common practice to introduce new objects into the ontology to account for the particular phenomena under investigation, e.g. degrees or extents in theories of gradable adjectives ([Cresswell 1976, Kennedy 1997] among others). However, by doing so one can easily miss the similarities that hold between constructions in different domains. For instance, typologists have established that many languages employ the same syntactic devices to encode spatial, temporal and comparative relations, and in some languages this even extends to causal relations [Stassen 1984]. One such language is Eskimo, which marks the reference object in locative and temporal expressions such as *out of his house* and *from the time you first started being here* with the ablative case. The ablative case is furthermore used to mark the cause in a causative construction such as *it was lost because*

[1]This paper is an extended abstract of [Faller 1998]. I would like to thank the members on my QP committee for their valuable comments, especially David Beaver.

it was so little. And in a comparative construction, the expression corresponding to the complement of *than* in English is marked with the ablative case, too [Mey 1976].

If the semantic relationships between these phenomena are to be studied, a framework that captures their similarities by using the same ontology is preferable to one that is only suitable within one domain. VSS, a framework in the tradition of Montague grammar with vectors as ontological primitives is a very promising framework in this respect. The present work shows that VSS can be extended to the domain of comparison, future work will have to show that it is also applicable to temporal and possibly causal relations.

In addition to giving a semantics for simple comparatives (1a.) and absolutes (1b.), I will present a compositional analysis of MPs in comparatives (*Sam is 3cm taller than Jo*) and of so-called comparatives of deviation, to be illustrated below.

While semantic theories of comparatives usually do include MPs on the representational level, it is often not made explicit how they can be incorporated compositionally.[2] The problem for many theories is that they require that an MP has access to the denotations of both the complement of *than* (*Jo* in (1)a.) and the matrix subject NP (*Sam* in (1)a.). Given the standard syntactic analysis of adjective phrases that is generally assumed ([MP [[more [ADJ]] XP]], where XP is the (extraposed) *than*-phrase), this is not straightforward. It will be shown that the VS semantics developed below for simple comparatives allows for a compositional incorporation of MPs, using the semantics [Zwarts 1995] and [Zwarts et. al 1997] present for MPs in locative PPs (*The tree is ten meters outside the house*).

Comparison of deviation is best illustrated with examples that do not allow a regular comparative interpretation. Consider (2). [3]

(2) Terry's RHR is lower than Jo's RHR is high.

(2) cannot be interpreted to mean that Terry's absolute RHR is lower than Jo's absolute RHR. Given the context that the RHR of an average person is between 68 and 72 beats per minute (bpm), and that a RHR below 68bpm is considered low, and a RHR above 72bpm high, (2) can only be interpreted to mean that the extent to which Terry's RHR is below the lower limit of the standard (68bpm) is greater than the extent to which Jo's RHR exceeds the upper limit (72bpm).[4] That is, what is being compared is the *deviation* of the two objects from the standards for the given dimension.

[2] Exceptions are [Bierwisch 1984, Hellan 1981, von Stechow 1984], which however do not account for comparison of deviation. [Kennedy 1997] presents a compositional analysis of MPs in absolutes, but not in comparatives

[3] RHR stands for "resting heart rate".

[4] I should mention that speakers' judgments on such examples vary greatly. The differences seem to have to do with how salient the standards are and how used a speaker is to dealing with such complex comparisons.

Most accounts of comparatives only mention comparison of deviation in passing, and at best a sketch of an analysis is presented [Kennedy 1997]. To my knowledge, the analysis presented below is the first attempt to give a compositional account of this phenomenon.

The remainder of the paper is structured as follows. Section 2 presents the VSS analysis of simple comparatives and absolutes, section 3 incorporates MPs and section 4 gives an account of comparison of deviation. Each of these sections first presents the semantics at an intuitive level before giving an example derivation. I conclude with section 5, which also briefly discusses the phenomenon of so-called cross-polar anomaly.

2 Analysis of Simple Comparatives and Absolutes

In VSS, vectors are ontological primitives. The operations vector addition $+$, scalar multiplication $\cdot$, and vector norm $|\,|$, are defined in the usual way (see appendix for definitions).

DAs make reference to dimensions such as HEIGHT. Each dimension is associated with a scale. A scale can be defined as a set of located vectors (see appendix).[5] A positive DA root such as *tall* denotes the set of vectors on the scale that point into the positive direction, a negative DA root such as *short* the set of vectors that point into the negative direction.

For the purposes of this paper, I assume that the compared objects are mapped onto their *eigenvector*, a positive vector on the appropriate scale located at the origin. Consider example (1)a. *Jo* is mapped onto the vector **j** in Fig. 1a. The comparison clause *than Jo* denotes the set of vectors V starting at the endpoint of **j**. **m** and **l** are two vectors in that set.

The comparative morphemes *more/-er* and *less* are analyzed as two-place functions that take a DA meaning and the set V as arguments, and map V onto a subset S, such that *more/-er* preserves the directionality of the vectors in the set denoted by the DA root, and *less* reverses it. That is, *shorter than X* denotes a set of vectors pointing into the negative direction, the vectors in *less short than X* point into the positive direction. For *tall*, it is just the opposite. Fig 1 a. illustrates *taller/less tall than Jo*, where $\mathbf{m} \in$ *taller than Jo*, $\mathbf{l} \in$ *less tall than Jo*.

Figure 1. Sam is taller than Jo

[5] Located vectors have fixed end- and startpoints (*epo*,*spo*) in contrast to vectors [Lang 1971], and are defined as pairs of points. Henceforth, I will use the term *vector* to mean located vector.

The subject of predication, *Sam*, is also mapped onto its *eigenvector*, $\mathbf{s}$, on the height scale. There will be exactly one vector $\mathbf{m}$ in the set denoted by *taller than Jo* such that its endpoint coincides with the endpoint of $\mathbf{s}$. This is illustrated in Fig. 1b. Based on this figure, the semantics for (1)a. will express that there exists a vector $\mathbf{m}$, such that $\mathbf{j} + \mathbf{m} = \mathbf{s}$.

The semantics for absolutes such as (1)b. is similar. Intuitively, a board is long if it exceeds a contextually determined standard $\mathbf{s}_l$, and short if it is below a standard $\mathbf{s}_s$, where $\mathbf{s}_l$ and $\mathbf{s}_s$ can be different. The standard thus corresponds to the complement of *than* in comparatives.[6] Accordingly, the semantics developed for (1)b. will express that there exists a vector $\mathbf{m}$ on the scale of LENGTH such that $\mathbf{s}_l + \mathbf{m} = \mathbf{b}$, where $\mathbf{b}$ is the eigenvector of *the board*.

Compositional Derivation. The following table shows which variables are conventionally used for which types in the remainder of the paper.[7]

variable	**v, w, u** etc.	**s**	W, V etc.	x, y, z etc.	p, q etc.
type	v, vector	s, stand.	vt, set of vect.	e, object	pt, point

The interpretations of the DA roots *tall* and *short* are:

(3) a. $\mathbf{tall'} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v})$
b. $\mathbf{short'} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge neg(\mathbf{v})$

HEIGHT is a predicate that holds of vectors in the height scale, *pos* and *neg* specify the directionality of their vectors arguments. The comparative morphemes have the following interpretations:

(4) a. $\mathbf{more'} \stackrel{\text{def}}{=} \lambda\text{DA}.\lambda\mathbf{w}.\lambda\mathbf{v}.\text{DA}(\mathbf{v}) \wedge epo(\mathbf{w}) = spo(\mathbf{v})$[8]
b. $\mathbf{less'} \stackrel{\text{def}}{=} \lambda\text{DA}.\lambda\mathbf{w}.\lambda\mathbf{v}.\text{DA}(\bar{\mathbf{v}}) \wedge epo(\mathbf{w}) = spo(\mathbf{v})$
where DA is the meaning of a dimensional adjective.

$\bar{\mathbf{v}}$ is defined as follows: Let $\mathbf{v}$ be the located vector $\langle p, q\rangle$, then $\bar{\mathbf{v}} = \langle q, p\rangle$. This effects the reversal of directionality. Applying $\mathbf{more'}/\mathbf{less'}$ to $\mathbf{tall'}$, we get the semantics for *taller*/*less tall*:

(5) a. $\mathbf{more'}(\mathbf{tall'}) = \lambda\mathbf{w}.\lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v}) \wedge epo(\mathbf{w}) = spo(\mathbf{v})$
b. $\mathbf{less'}(\mathbf{tall'}) = \lambda\mathbf{w}.\lambda\mathbf{v}.\text{HEIGHT}(\bar{\mathbf{v}}) \wedge pos(\mathbf{v}) \wedge epo(\mathbf{w}) = spo(\mathbf{v})$
$\Leftrightarrow \lambda\mathbf{w}.\lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge neg(\mathbf{v}) \wedge epo(\mathbf{w}) = spo(\mathbf{v})$

[6] I will not discuss how these standards are derived, see [Kennedy 1997] for discussion.

[7] To have a different type for standard vectors is for mere convenience, and has at this point no theoretical motivation.

[8] I will assume throughout the paper that *-er* and *more* get the same interpretation.

As mentioned above the compared objects are mapped onto their *eigenvectors*. This is done with a function *dim*, which is identified with the interpretation of *than*: $\mathbf{than'} \stackrel{\text{def}}{=} dim$. *taller than Jo* then denotes:

(6) $\mathbf{more'}(\mathbf{tall'})(\mathbf{than'}(\mathbf{Jo})) = \lambda\mathbf{v}.\textsc{height}(\mathbf{v}) \wedge pos(\mathbf{v}) \wedge epo(dim(Jo)) = spo(\mathbf{v})$

The next step in the compositional process has to be to predicate this expression of the subject, *Sam* in the running example. This is not directly possible, since *taller than Jo* denotes a set of vectors, i.e. we first have to map the subject onto its *eigenvector*. I introduce a "anti-dimension" function dim^- of type $\langle vt, \langle e, t\rangle\rangle$, which returns the set of objects, whose *eigenvectors* have the same endpoints as the vectors in the set denoted by a comparative DA plus *than*-clause. This function is parallel to loc^- used by [Zwarts et. al 1997] to type-lift the subject of locative PPs.

(7) $dim^- \stackrel{\text{def}}{=} \lambda W \lambda x[\exists\mathbf{v}[W(\mathbf{v}) \ \wedge \ epo(\mathbf{v}) = epo(dim(x)) \ \wedge \ |\mathbf{v}| > 0]]$

Neither loc^- nor dim^- are associated with a particular lexical item. Instead, their application is triggered by the type mismatch between the subject of predication and the predicate. At this point one might ask why we need these functions at all, since one could just as well interpret the comparative morphemes as relations between vectors and have them type-lift both objects. I will show in section 3 that this complication is necessary to incorporate MPs compositionally. Applying dim^- to **taller than Jo′** and the result to **Sam′** gives us:

(8) $\exists\mathbf{v}[\textsc{height}(\mathbf{v}) \ \wedge \ pos(\mathbf{v}) \ \wedge \ epo(dim(Jo)) = spo(\mathbf{v}) \ \wedge \ epo(\mathbf{v}) = epo(dim(Sam)) \ \wedge \ |\mathbf{v}| > 0]$

This is a fairly complicated expression, but it can be simplified somewhat by replacing the defining points of $\mathbf{v}$ in the representation with just $\mathbf{v}$ and expressing the relation between the three relevant vectors using vector addition. We can also abbreviate $dim(x)$ with a vector constant. What (8) amounts to is (9)a. (9)b. shows he semantic representation for *Sam is shorter than Jo*.

(9) a. $\exists\mathbf{v}[\mathbf{s} = \mathbf{j} + \mathbf{v} \ \wedge pos(\mathbf{v}) \wedge \textsc{height}(\mathbf{v}) \ \wedge \ |\mathbf{v}| > 0]$
b. $\exists\mathbf{v}[\mathbf{s} = \mathbf{j} + \mathbf{v} \ \wedge neg(\mathbf{v}) \wedge \textsc{height}(\mathbf{v}) \ \wedge \ |\mathbf{v}| > 0]$

As mentioned above, absolute DAs compare their subject with a contextually determined standard. Following [Kennedy 1997], I assume that this standard is introduced by a phonologically empty absolute morpheme $\emptyset$. The interpretation for $\emptyset$ is as follows:

(10) $\emptyset' \overset{\text{def}}{=} \lambda \text{DA} \lambda \mathbf{v}.\text{DA}(\mathbf{v}) \;\wedge\; \mathit{epo}(\mathbf{s}) = \mathit{spo}(\mathbf{v})$

The derivation of an absolute construction proceeds exactly parallel to that of a comparative, and the result for the example (1)c. is:

(11) $\exists \mathbf{v}[\mathbf{b} = \mathbf{s_l} + \mathbf{v} \;\wedge \mathit{pos}(\mathbf{v}) \wedge \textsc{length}(\mathbf{v}) \;\wedge\; |\mathbf{v}| > 0]$

3 Incorporating MPs

This section discusses sentences of the form:

(12) a. Sam is 3cm taller than Jo. b. The board is 100cm long.

As illustrated in the previous section, we have a denotation for the phrase *taller than Jo* before dim^- applies, namely the set of vectors S. Now, unlike in relational accounts, there is no need to access the denotation of the subject in order to calculate the difference between the compared objects. The vectors in S denote that difference. An MP can now simply be defined as a function that applies to S and maps it onto a subset of vectors which have a specific length, *3cm* in (12)a.

For absolutes with MPs matters are a bit more complicated, since there is no reference to a standard. A board that is 100cm long is not necessarily long compared to $\mathbf{s}_l$. Thus, in the presence of an MP we will either have to say that the standard is somehow set to zero, or that in that case the absolute morpheme does not introduce a standard at all. In either case, the MP will in effect end up specifying the length of the *eigenvector* of the subject.

A further complication is that only positive adjectives can be modified by MPs in the absolute:

(13) a. The board is 100cm long b. # The board is 100cm short.

Applying an MP to a negative adjective where the standard is zero, will require there to be a vector $\mathbf{v}$ which starts at the origin and points into the negative direction. For scales without a negative extension such as the scale of length, there is no such vector. This in itself might be enough to explain the anomaly of (13)b. But we can also cause the semantic composition to fail by giving negative adjectives an interpretation that requires the standard $\mathbf{s}$ to be greater than $\mathbf{0}$. This captures better the observation that negative adjectives are always used with reference to a standard. For example, some speakers of English report that *The board is 100cm short* is acceptable, but that it can only mean that the board is 100cm *too* short with respect to some standard.

Compositional Derivation. The interpretation for an MP such as $3cm$ as given by [Zwarts et. al 1997] is:

(14) $\mathbf{3cm}' \stackrel{\text{def}}{=} \lambda W.\lambda \mathbf{v}.W(\mathbf{v}) \ \wedge \ |\mathbf{v}| = 3cm$

In the derivation of (12)a., $\mathbf{3cm}'$ can be directly applied to the set denoted by *taller than Jo*. The semantic representation for (12)a. with the MP is:

(15) $\exists \mathbf{v}[\mathbf{m} = \mathbf{c} + \mathbf{v} \wedge \text{HEIGHT}(\mathbf{v}) \ \wedge \ |\mathbf{v}| = 3cm]$

As hinted at above, the integration of MPs in absolutes poses some problems. One can either introduce a second interpretation for the absolute morpheme that does not introduce a standard[9], but simply requires that $\mathbf{v}$'s startpoint is $\mathbf{0}$:

(16) $\emptyset' \stackrel{\text{def}}{=} \lambda \text{DA} \lambda \mathbf{v}.\text{DA}(\mathbf{v}) \ \wedge \ spo(\mathbf{v}) = \mathbf{0}$

This is compositional, the MP can directly apply to the set denoted by *tall-∅* and specify the length of $\mathbf{v}$ as with comparatives. However, this predicts that (12)b. is ambiguous, which it is clearly not. Alternatively, one can let the presence of an MP force the value of $\mathbf{s}$ to be $\mathbf{0}$. An anonymous reviewer questioned whether this could be done compositionally, and indeed I have not yet succeeded in making this compositional.[10]

I will leave this area for future research, but will briefly discuss in section 5 what predictions the second approach makes with respect to regular subdeleted comparative clauses.

[9] This is essentially what [Kennedy 1997] does.

[10] A first attempt is the following modification of the MP interpretation:

(17) $\mathbf{3cm}' \stackrel{\text{def}}{=} \lambda W.\lambda \mathbf{v}.W(\mathbf{v}) \ \wedge \ |\mathbf{v}| = 3cm \ \wedge \ [\forall \mathbf{s}(spo(\mathbf{v}) = epo(\mathbf{s})) \rightarrow \mathbf{s} = \mathbf{0}]$

What the added implication says is that if the vectors $\mathbf{v}$ in S are the result of mapping a standard $\mathbf{s}$ onto the vectors starting at its endpoint, then this standard is to be set to zero. This assumes that the pragmatic component only supplies values for variables that have not been supplied by the semantic component. However, as David Beaver pointed out, this does not work for examples like the following:

(i) Jo is 7 feet tall and Sam is tall too.

If we set the standard introduced by the first occurence of *tall-∅* to zero, then the standard introduced by the second occurence is zero, too, since they are necessarily the same. What is needed is a way to set the standard variable to zero locally only.

Another approach is [Bierwisch 1984]. He deals with the problem by means of conditions on semantic constants. In his account, an MP, which denotes an interval on a scale, is itself an argument of a concatenation operator +, the other argument is a variable N_C over intervals (comparable to $\mathbf{s}$). N_C can either be $= 0$ or > 0. He then imposes a condition on '+' which says that if the first argument is an MP, then $\text{N}_C = 0$, otherwise $\text{N}_C > 0$ (the exact value is contextually determined). Thus, he essentially allows the type of one argument of a function to determine the value of another argument, which is a rather unusual move. Clearly, more research is required to solve this problem.

4 Comparison of Deviation

As should be clear from the paraphrase given above for (2), repeated here as (18), in comparatives of deviation, one first has to calculate the deviation vectors of the compared objects from the standards, and then compare their length.

(18) Terry's RHR is lower than Jo's RHR is high.

For this, the two vectors themselves are mapped onto their *eigenvectors*, $\mathbf{v}'$ and $\mathbf{w}'$, on a scale of length. This mapping will be part of the function denoted by the comparative morphemes in comparison of deviation constructions. The relations between the vectors and scales involved are illustrated in Fig. 2 (where **sl**/**sh** is the standard for a low/high RHR, and **t** and **j** are the *eigenvectors* for Terry's and Jo's RHRs respectively).

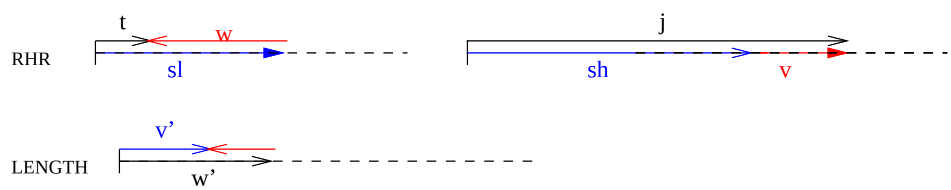


Figure 2: Terry's RHR is lower than Jo's RHR is high.

Compositional derivation. Before giving the semantics for comparison of deviation, I have to say a few words about the interpretation of subdeleted *than*-clauses in general, though due to limitations of space, I cannot give a full semantics (but see [Faller 1998]). Subdeleted comparative clauses are characterized by the fact that they cannot contain an overt MP, and it is generally assumed that the *than*-clause contains a gap of some sort. Following [Kennedy 1997] and others, I assume that the *than*-clause is a definite description, and will use the ι operator to express this. The denotation of *than Jo's RHR is high* in (18) is:

(19) $\iota\mathbf{v}[\mathrm{RHR}(\mathbf{v}) \ \wedge pos(\mathbf{v}) \ \wedge \ \mathbf{sh} + \mathbf{v} = dim(Jo's\ RHR)]$

This is a description of a vector $\mathbf{v}$ which corresponds to the deviation of *Jo's RHR* from the standard **sh**. It is derived by using the semantics for the absolute morpheme and DA roots above, but instead of using dim^- to predicate *high* of *Jo's RHR*, which would give us a sentence, the following function dim^-_{dev} denoted by *than* is applied:

(20) $than_{dev} : dim^-_{dev} \stackrel{\text{def}}{=} \lambda x.\lambda W.\iota\mathbf{v}[W(\mathbf{v}) \ \wedge \ epo(\mathbf{v}) = epo(dim(x))]$

This function takes an individual x and a set of vectors W as arguments, and outputs a definite description of a vector which is in W and whose endpoint is equal to the endpoint of the *eigenvector* of x.[11]

The comparative morphemes in the main clause of a comparative of deviation get the following interpretation:[12]

(21) $\mathbf{more'_{dev}} \stackrel{\text{def}}{=} \lambda \text{DA}.\lambda \mathbf{w}.\lambda \mathbf{v}.\text{DA}(\mathbf{v}) \ \wedge \ epo(\mathbf{s}) = spo(\mathbf{v}) \wedge$
$\exists \mathbf{u}[dim(\mathbf{w}) + \mathbf{u} = dim(\mathbf{v}) \ \wedge \ pos(\mathbf{u}) \wedge \text{LENGTH}(\mathbf{u})]$

Like the regular comparative morphemes, $more_{dev}$ and $less_{dev}$ take a gradable adjective DA and a vector **w** as their arguments, and output a set of vectors. In addition, they existentially introduce a vector **u**, the difference vector calculated on the scale of LENGTH between dim(**w**) and dim(**v**), the *eigenvectors* of **u** and **v**. Furthermore, they introduce the standard **s** for the DA in the matrix clause. After applying $more_{dev}$ to *low* and *than Jo's RHR is high*, and then dim^- to the result and to the subject *Terry*, we get the following semantics for (18):

(22) $\exists \mathbf{v}.\text{RHR}(\mathbf{v}) \wedge neg(\mathbf{v}) \wedge \mathbf{sl}+\mathbf{v} = dim(Terry) \wedge \exists \mathbf{u}[dim(\mathbf{dev\text{-}jo})])+\mathbf{u} =$
$dim(\mathbf{v}) \ \wedge \ pos(\mathbf{u}) \wedge \text{LENGTH}(\mathbf{u})]$
where **dev-jo** abbreviates *than Jo's RHR is high*

This is the desired result. It says that there is a vector **v** which denotes the deviation of Terry's RHR from the standard of low RHRs, and that there is a vector **w** which denotes the difference between the length of **v** and the length of the vector **dev-jo**, which is the deviation of Jo's RHR from the standard of high RHRs.

5 Conclusion and Discussion

The previous sections have demonstrated that VSS can quite straightforwardly be applied to adjectival comparatives. It is thus worthwhile to try to adapt VSS to account for temporal and possibly causal relations as well. In addition to the phenomena discussed, I would like to briefly address two issues that arise in connection with regular subdeleted comparatives, the VS semantics of which I could not present within the scope of this paper. The first arises from the discussion of comparison of deviation. The example I derived, (18), has, it is claimed, no regular comparative interpretation. The question that presents itself is why? This interpretation is not generally ruled out, witness (23).

[11] Note that the vector set argument of dim^-_{dev} is not restricted to be the output of the absolute morpheme. That is, the semantics allows sentences like *The board is longer than the table is longer than the desk is wide.* At this point, I do not want to take a stance on whether or not such sentences are ungrammatical or simply very hard to process.

[12] $\mathbf{less'_{dev}}$ is just like $\mathbf{more'_{dev}}$ with $neg(\mathbf{u})$ instead of $pos(\mathbf{u})$

(23) The board is longer than the table is wide.

In fact, the first interpretation that comes to mind, is the regular one: the absolute extent of the board along the LENGTH-dimension is greater than the absolute extent of the table along the WIDTH-dimension. Only after some amount of persuasion do native speakers agree that it can also have the deviation reading. [Kennedy 1997], who introduces the term *cross-polar anomaly* for sentences like (18), explains the its infelicity with respect to the regular interpretation in terms of the opposite polarity of the two adjectives involved, *high* is a positive adjective, *low* is a negative adjective. In his theory, positive and negative adjectives map onto different types of extents on a scale, and the infelicity arises from the incommensurability of negative and positive extents. In contrast, I would like to argue, following [Bierwisch 1984], that the regular reading should be ruled out on the basis of Gricean-style principles such as *Don't be redundant*. Since the two adjectives in (18) make reference to the same scale, the second one is redundant unless it provides some additional information such as the second standard needed for comparison of deviation. Support for this approach comes from examples like the following (taken from [Rusiecki 1984]):

(24) This swimming pool is shorter than that one is wide.

(24) too contains a negative and a positive adjective, but the regular reading is not ruled out. The only difference between (24) and (18) is that the two adjectives in (24) make reference to two different scales. Thus, the second adjective in (24) is not redundant, even under a regular interpretation, and it is therefore felicitous.

The second issue has to do with the contrast between (24) and (25).

(25) This swimming pool is wider than that one is short.

Some native speakers of English find (25) much less acceptable than (24) [Rusiecki 1984]. A tentative explanation is the following. Recall that the *than*-clause contains a gap in the MP position. We can therefore assume, as other theories do, that the gap is a phonologically empty MP-morpheme. If we adopt a semantics in which the presence of an MP sets the standard variable introduced by the absolute morpheme to zero, the this empty MP will do exactly that. But the semantics of a negative DA root requires $\mathbf{s} > 0$ (see section 4). Thus, the derivation fails.

In conclusion, I hope to have shown that VSS is well suited as a basis for future empirical work both in terms of its predictions regarding comparatives, and in terms of its ability to draw together semantic analyses of a range of different empirical phenomena.

6 Appendix

The definitions of a vector space, vector domain, scalar product and metric are slightly adapted from [Zwarts et. al 1997] with the kind permission of Yoad Winter.

A *vector space* over the field of real numbers $\mathbf{R}$ is a quadruple $\langle V, 0, +, \cdot \rangle$ s.t. V is a set, $0 \in V$ (the zero vector) and the functions $+ : (V \times V) \rightarrow V$ (vector addition) and $\cdot : (\mathbf{R} \times V) \rightarrow V$ (scalar multiplication) satisfy for all $u, v, w \in V$ and $s, r \in \mathbf{R}$:
1. $(u + v) + w = u + (v + w)$ [associativity] 2. $0 + v = v + 0 = v$
3. There is an element $-v \in V$ s.t. $v + (-v) = 0$ [inverse]
4. $u + v = v + u$ [commutativity] 5. $s(u + v) = su + sv$
6. $(s + r)v = sv + rv$ 7. $(sr)v = s(rv)$ 8. 1v=v (1 is the unit element of $\mathbf{R}$)

Definition (the vector domain). Let $\langle V, 0, +, \cdot \rangle$ be a vector space over $\mathbb{R}$ with f a positive scalar product and $w \in V$. We define:

$V_w \stackrel{\text{def}}{=} \{\langle w, v\rangle : v \in V\}$ $\quad 0_w \stackrel{\text{def}}{=} \langle w, 0\rangle$

For all $u, v \in V$: $\langle w, u\rangle +_w \langle w, v\rangle \stackrel{\text{def}}{=} \langle w, u + v\rangle$

For all $s \in \mathbb{R}$, $v \in V$: $s \cdot_w \langle w, v\rangle \stackrel{\text{def}}{=} \langle w, s \cdot v\rangle$

For all $u, v \in V$: $f_w(\langle w, u\rangle, \langle w, v\rangle) \stackrel{\text{def}}{=} f(u, v)$

For every $w \in V$:$\langle V_w, 0_w, +_w, \cdot_w\rangle$ is a vector space over $\mathbb{R}$ with f_w a positive scalar product, which determines a norm denoted by $|\ |_w$. Trivially, the domain D_v is equal to the union of vector spaces $\cup_{w \in V} V_w$.

A *scalar product* over a vector space V is a function $f : (V \times V) \rightarrow \mathbf{R}$ that satisfies for all $u, v, w \in V$, $s \in \mathbf{R}$: 1. $f(v, w) = f(w, v)$ [commutativity]
2. $f(u, v + w) = f(u, v) + f(u, w)$ [distributivity over +]
3. $f(sv, w) = sf(v, w) = f(v, sw)$ [distributivity over$\cdot$]

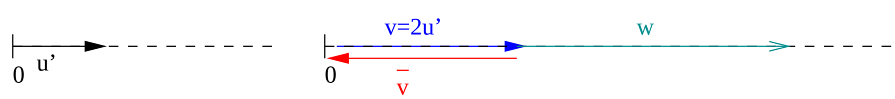
A scalar product is called *positive* iff for every $v \in V$: $f(v, v) \geq 0$ and for every $v \in V \setminus \{0\}$: $f(v, v) > 0$. For a positive scalar product f the *norm* of a vector $v \in V$ is denoted $|v| = \sqrt{f(v, v)}$. For any set X a *metric* for X is a non-negative function $d : (X \times X) \rightarrow \mathbf{R}^+$ that satisfies for all $x, y, z \in X$:
1. $d(x, y) = d(y, x)$ 2. $d(x, y) + d(y, z) \geq d(x, z)$ 3. $d(x, y) = 0$ iff $x = y$

The elements $\mathbf{v}$ in a vector space V_w as defined above are ordered pairs of points $\langle w, v\rangle$, where w is the center of V_w. An ordered pair of points is called a *located* vector [Lang 1971]. The vectors in V_w are located in $\mathbf{w}$. Vectors that are located at the origin, are uniquely determined by their endpoint. For the purposes of this paper, I find it more convenient to also use located vectors whose startpoints differ from the origin. Addition of two located vectors $\mathbf{v}$, $\mathbf{w}$, where *epo*($\mathbf{v}$) = *spo*($\mathbf{w}$) is defined as [Kowalsky 1967]:

$\langle p, q\rangle +_l oc\langle q, r\rangle = \langle p, r\rangle$

This is the only kind of vector addition used in the paper, and I will therefore drop the subscript *loc*.

I define a *scale* as a set of vectors $S \subset V_0$, where V_0 is a vector space with $\mathbf{0}$ as its origin. Let m be a metric for V_0 and $\mathbf{u}$ a vector in V_0 such that $|\mathbf{u}|$=1m. Then, $\mathbf{u}$ is the *unit vector* for V_0. Let $\mathbf{u}'$ be a located unit vector $\mathbf{u}' = \langle 0, 1\rangle$. Then, S is inductively defined as follows: Base Case: $\mathbf{u}'$. Inductive Steps. All located vectors $\mathbf{v}$ such that $\mathbf{v} = s \cdot \mathbf{u}'$, for all $s \in \mathbb{R}^+$. For any located vector $\mathbf{v} = \langle p, q\rangle$, a located vector $\mathbf{w} = \langle q, r\rangle$ s.t. $\langle w, r\rangle = \langle w, s\rangle$ for all $s \in \mathbb{R}^+$. For any located vector $\mathbf{v} = \langle p, q\rangle$, a located vector $\bar{\mathbf{v}} = \langle q, p\rangle$



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