Summarising Data

Mark Lunt

Centre for Epidemiology Versus Arthritis
University of Manchester

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Summarising Data

Today we will consider

- Different types of data
- Appropriate ways to summarise these data
  - Graphical Summary
  - Numerical Summary

Types of Data

Qualitative	Nominal	Outcome is one of several categories
	Ordinal	Outcome is one of several ordered categories
Quantitative	Discrete	Can take one of a fixed set of numerical values
	Continuous	Can take any numerical value

Examples of Types of Data

Nominal	Blood group; Hair colour.
Ordinal	Strongly agree, agree, disagree, strongly disagree.
Discrete	Number of children.
Continuous	Birthweight.

Caveats with Data Types

- Distinction between nominal and ordinal variables can be subjective: e.g. vertebral fracture types: Wedge, Concavity, Biconcavity, Crush.
  Could argue that a crush is worse than a biconcavity which is worse than a concavity . . . , but this is not self-evident.
- Distinction between ordinal and discrete variables can be subjective: e.g. cancer staging I, II, III, IV: sounds discrete, but better treated as ordinal.
- Continuous variables generally measured to a fixed level of precision, which makes them discrete. Not a problem, provide there are enough levels.

Types of Variables

What type of variable are each of the following:

- Number of visits to a G.P. this year
- Marital Status
- Size of tumour in cm
- Pain, rated as minimal/moderate/severe/unbearable
- Blood pressure (mm Hg)

Summarizing Qualitative Data

- Count the number of subjects in each group.
- The count is commonly refered to as the *frequency*
- The proportion in each group is referred to as the *relative frequency*
- Stata command to produce a tabulation is `tabulate` *`varname`*

Numerical Summary of Qualitative Data

region	Freq.	Percent	Cum.
Canada	422	22.84	22.84
USA	541	29.27	52.11
Mexico	223	12.07	64.18
Europe	493	26.68	90.85
Asia	169	9.15	100.00
Total	1,848	100.00	

Graphical Summary of Qualitative Data

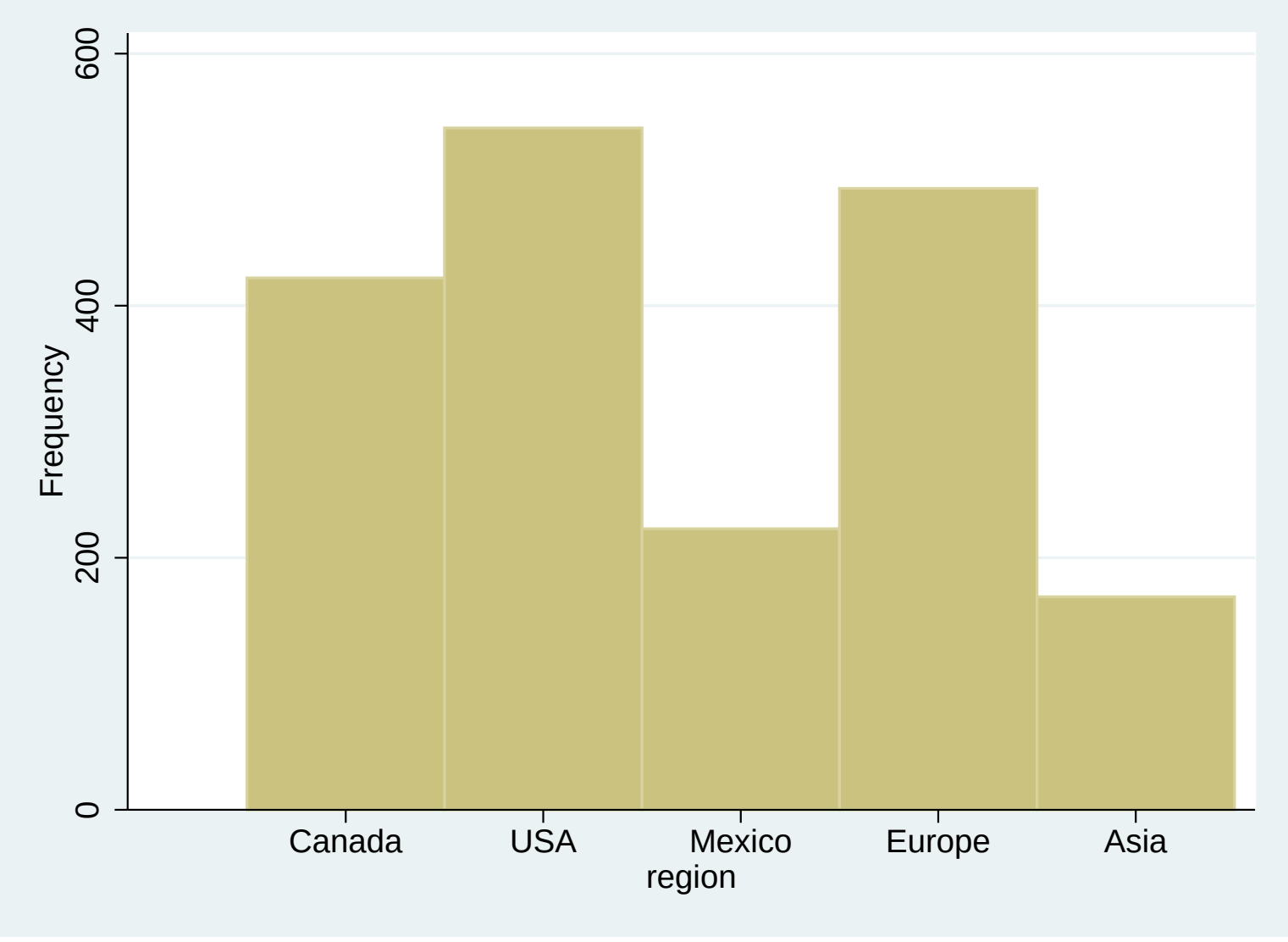
Bar Chart: Data represented as a series of bars, height of bar proportional to frequency.

Pie Chart: Data represented as a circle divided into segments, area of segment proportional to frequency.

Pictograms: Similar to bar chart, but uses a number of pictures to represent each bar.

Bar chart is the easiest to understand.

Bar Chart



Summarizing Quantitative Data

Simplest method: treat as qualitative data.

- Divide observations into groups
  - May be unnecessary for discrete data.
- Look at the frequency distribution of these groups
- Can use table or diagram.

The Histogram

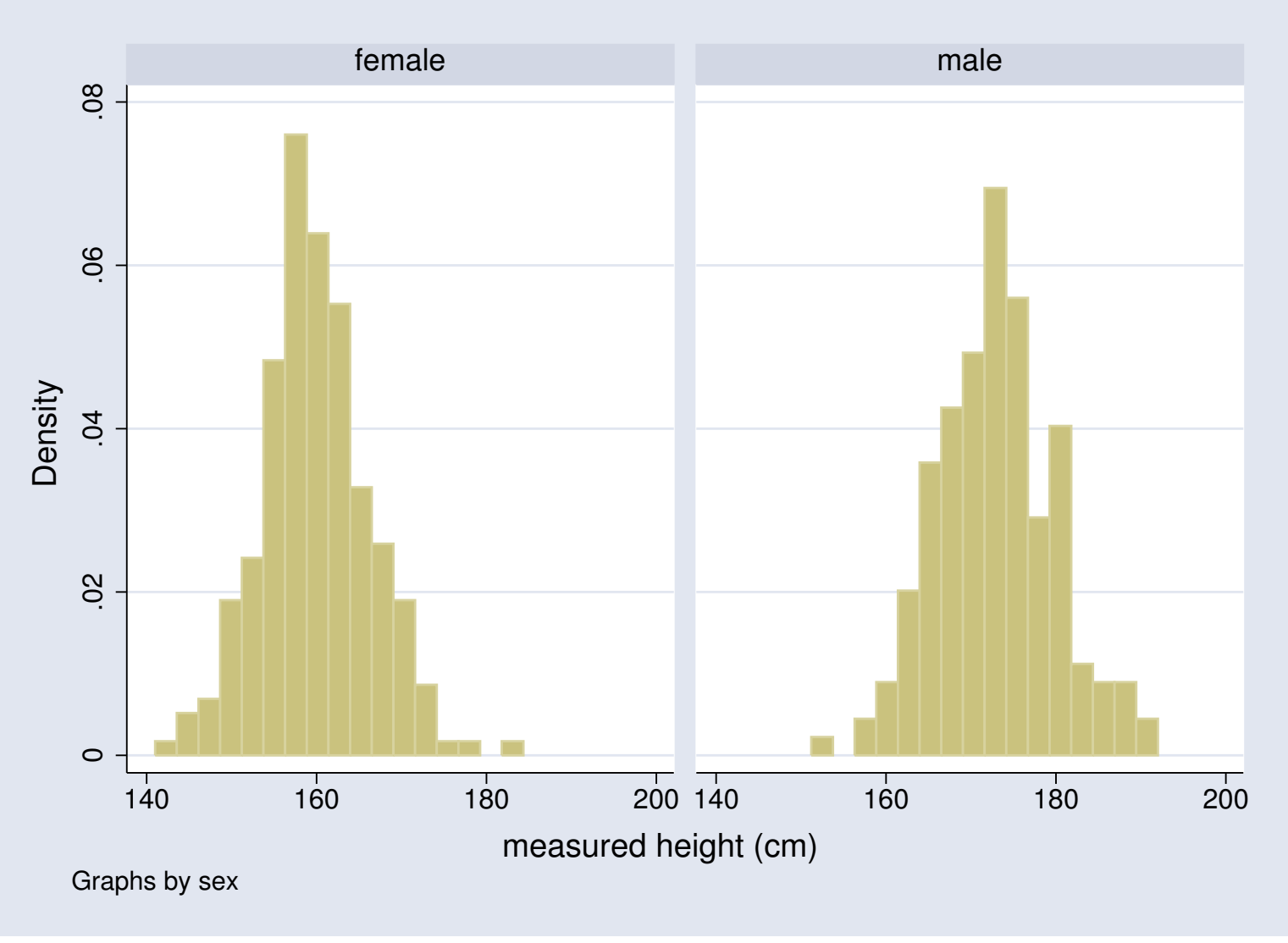
- Similar to a bar chart
- Continuous, not categorical variable
- Area of bars proportional to probability of observation being in that bar
- Axis can be
  - Frequency (heights add up to n)
  - Percentage (heights add up to 100%)
  - Density (*Areas* add up to 1)

How Many Groups ?

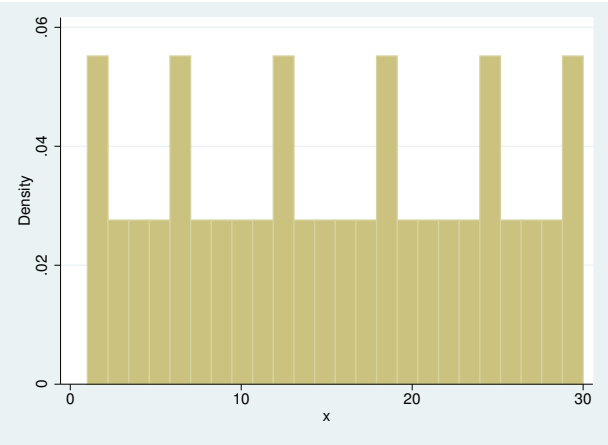
Impossible to say.

- Depends on the number of observations: if individual groups are too small, results are meaningless.
- With discrete variables, exact positions of boundaries may be important.
- Tables need few groups, graphs can have more if sufficient numbers.
- May be decided for you in software.

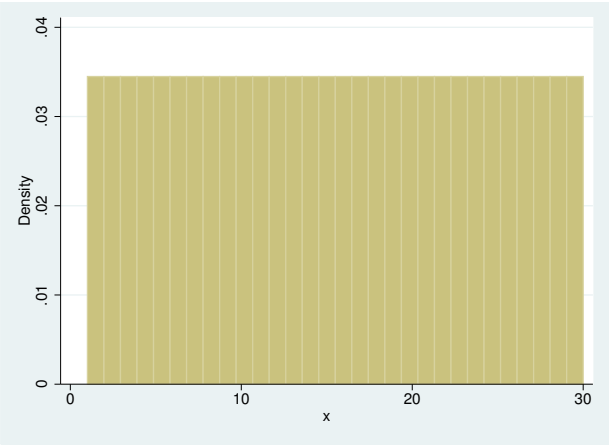
Histograms


female
male
Density
.08
.06
.04
.02
0
140
160
180
200
measured height (cm)
Graphs by sex

Histogram: Effect of Wrong number of bins



24 bins (default)



30 bins (correct)

Bar charts and histograms in Stata

- `histogram` *`varname`* produces a histogram
- Number of bars can by set by option `bin()`
- Width of a bar can be set by option `width()`
- `histogram` *`varname`*`, discrete` produces a bar chart
- What stata calls a bar chart is the mean of second variable subdivided by category, rather than a frequency.

Numerical Summary of Quantitative Data

- Need to know:
  1. What is a typical value (“location”)
  2. How much do the values vary (“scale”)
- Simplest distribution to summarize is the normal distribution
- Other summary statistics (skewness, kurtosis etc) thought of relative to normal distribution.

Measures of Location

What is the value of a “typical” observation ? May be:

- (Arithmetic) Mean
- Median
- Other forms of mean
  - Rarely used
  - Only if data has been transformed

Arithmetic Mean

"Add them up and divide by how many there are."

$$\begin{aligned} \bar{x} &= \frac{x_1 + x_2 + \ldots + x_n}{n} \\ &= (\Sigma_{i=1}^{n} x_i)/n \end{aligned}$$

Median

"Arrange in increasing order, pick the middle." If an even number of observations, take mean of middle two.

- Ignores the precise magnitude of most observations
  - Contains less "information" than mean
  - May be useful if there are outliers
- Less easy to use mathematically.

Mean vs. Median

Consider this series of durations of absence from work due to sickness (in days).
1,1,2,2,3,3,4,4,4,4,5,6,6,6,6,7,8,10,10,38,80
Mean = 10
Median = 5
Very few observations are as large as the mean: median is more “typical”.

Percentiles

- The x^{th} percentile is the value than which x% of observations are smaller and $(100 - x)$% are larger.
- The median is the 50th percentile.
- Other centiles can easily be calculated, eg 5th, 25th etc.

Measures of Variation

How close to the “typical” value are other values.

- Range
- Inter-quartile range
- Variance

Simple Measures of Variation

Range

- (Largest measurement) - (smallest measurement)
- Depends on only two measurements
- Can only increase as you add more to the sample

Inter-quartile Range

- (75th centile) - (25th centile).
- Less sensitive to extreme values
- Need fairly large numbers of observations

Standard Deviation

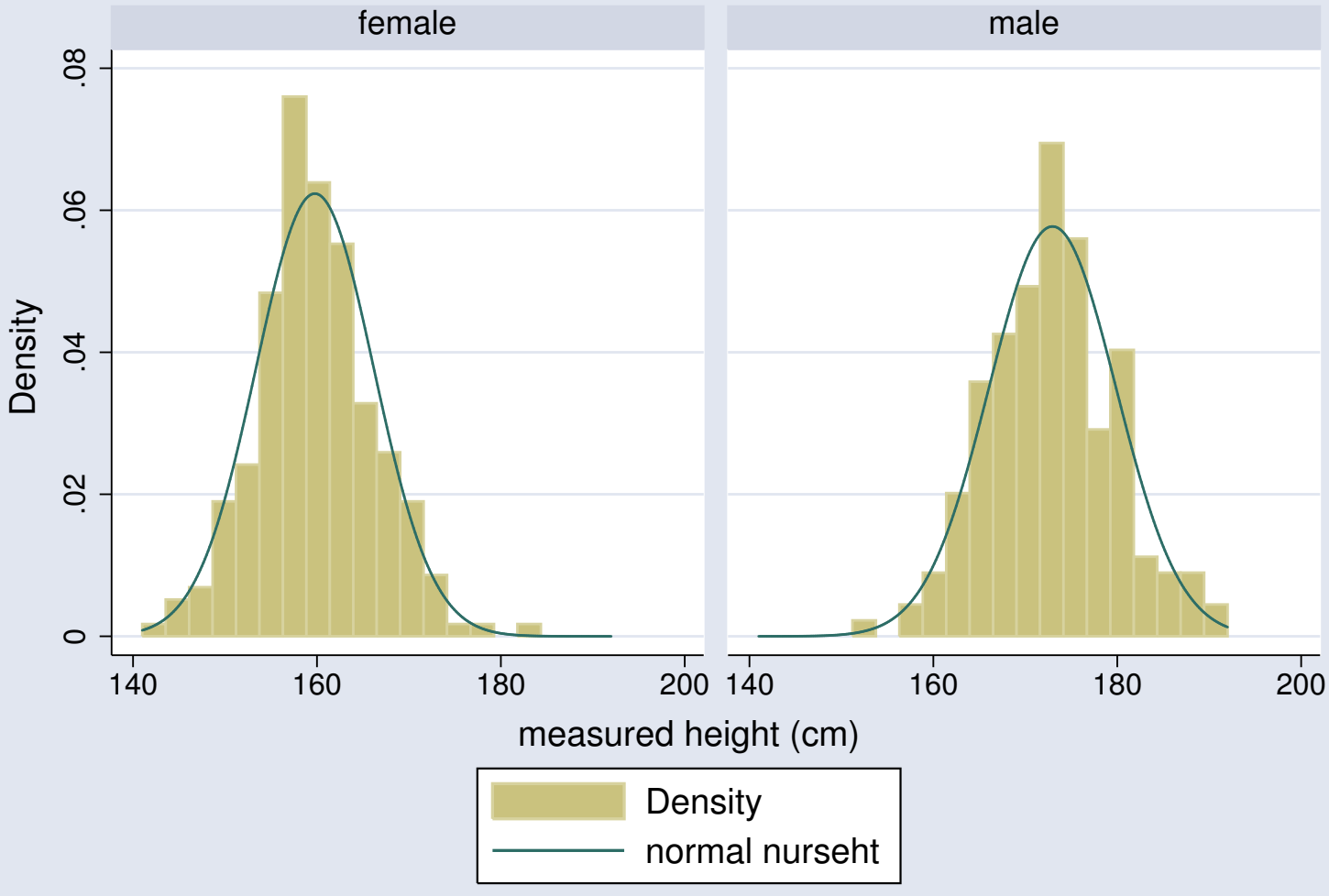
$$\text{Standard Deviation} = \sqrt{\Sigma(x_i - \bar{x})^2/n}$$

- **Nearly** the average difference from the mean
- Uses information from every observation
- Not robust to outliers
- Variance is easy to use mathematically
- Standard deviation is the same units as the observations

The Normal Distribution

- Symmetrical “Bell-shaped” distribution
- Easiest to use mathematically
- Many variables are normally distributed
- Can be described by two numbers
  - Mean (measure of location)
  - Standard Deviation (measure of variation)

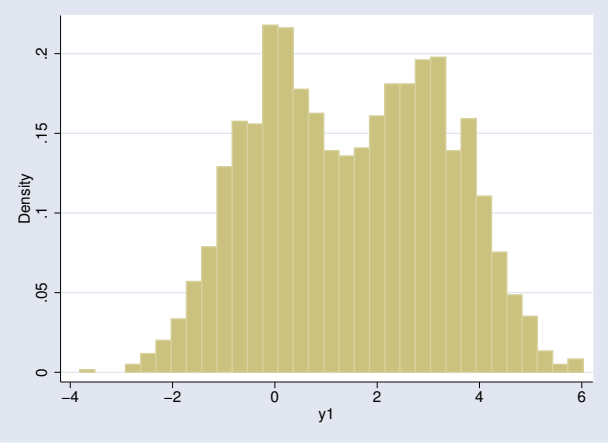
Histogram & Normal Distribution



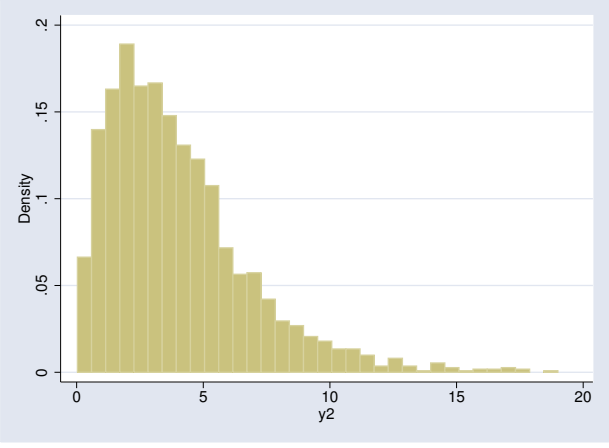
Non-Normal Distributions

- Normal distribution is symmetric.
- Asymmetric distributions are called “skewed”:
  - Positively skewed = some extremely high values (mean $>$ median).
  - Negatively skewed = some extremely low values (mean $<$ median).
- Distribution may have more than one “peak”: bi-modal.
  - Usually formed by mixing two different groups.

Non-Normal Distributions



Bimodal Distribution



Positively Skewed Dist'n

Summary Statistics in Stata

- `summarize` *`varlist`* will give mean, SD, min and max
- `summarize` *`varlist`*`, detail` also gives percentiles
- `tabstat` or `table` can produce tables of summary statistics

Numerical Summary: Table 1

- Quantitative variables
  - Need a measure of location & variation
    - Normal variables: mean and SD
    - Skewed variables: median and IQR
  - Need to give units
- Qualitative variables
  - Number and % in each category

Numerical Summary Example

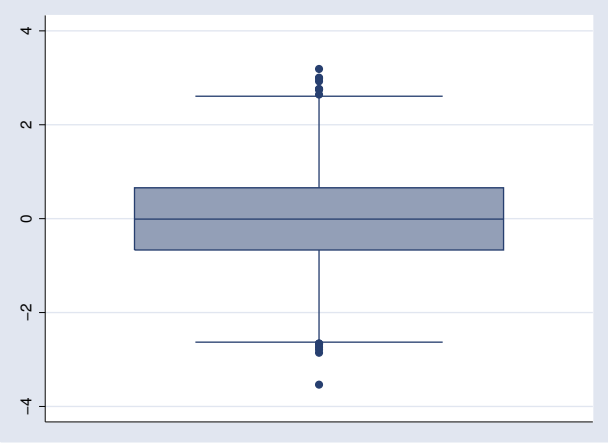
Age in years: Mean (SD)		63 (7.9)
Spine BMD in g/cm^2: Median (IQR)		1.05 (0.78, 1.30)
Gender: n (%)	Male	1537 (44)
	Female	1924 (56)

The Box and Whisker Plot

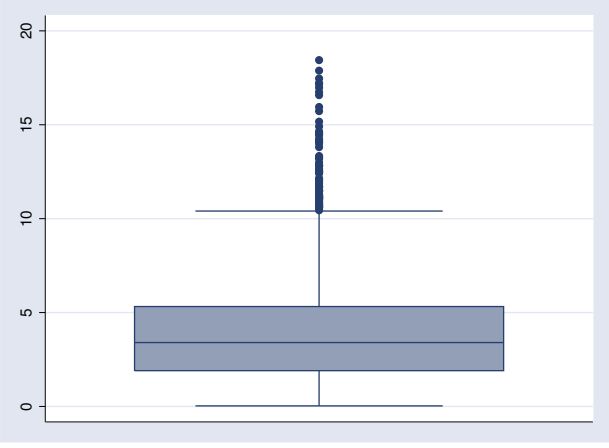
Very efficient summary of distribution:

- Shows median, upper and lower quartiles (25th and 75th percentiles).
- Also shows range of “normal” values and individual “unusual” values.
- Definitions of “normal” and “unusual” differ.
- Will demonstrate skewness, not bimodality.
- Stata command: `graph box` *`varname`*`, [by(`*`groupname`*`)]`

Box and Whisker Plots



Normal Distribution



Positively Skewed Dist'n

Transforming Data

- Skewed distributions may be made symmetric by a transformation.
- Taking logs is the most common.
- Other transformations (e.g. square root, reciprocal) can be used, but can be very difficult to interpret.
- May be better to transform back to original units to present results.
    - Geometric mean is back-transformation of mean of log-transformed data.

Further Reading

- **Edward R. Tufte**, *The Visual Display of Quantitative Information* was the classic text on statistical graphs.
- Huge data visualisation industry now