Statistical Modelling in Stata: Categorical Outcomes

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Categorical Outcomes

- Nominal
- Ordinal

Nominal Outcomes

- Categorical, more than two outcomes
- No ordering on outcomes

R by C Table: Example

	Females		Males		Total	
Indemnity	234	(51%)	60	(40%)	294	(48%)
Prepaid	196	(42%)	81	(53%)	277	(45%)
No Insurance	32	(7%)	13	(8%)	45	(7%)
Total	462	(100%)	154	(100%)	616	(100%)

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```
tab insure male, co chi2
```

Analysing an R by C Table

- χ^2-test: says if there is an association
- Need to assess what that association is
- Can calculate odds ratios for each row compared to a baseline row

Odds Ratios from Tables

	Females	Males	Total
Indemnity	234	60	294
Prepaid	196	81	277
No Insurance	32	13	45
Total	462	154	616

- Prepaid vs Indemnity
  - OR for males = $\frac{81 \times 234}{60 \times 196} = 1.61$
- No Insurance vs Indemnity
  - OR for males = $\frac{13 \times 234}{60 \times 32} = 1.58$

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Multiple Logistic Regression Models

- Previous results can be duplicated with 2 logistic regression models
  - Prepaid vs Indemnity
  - No Insurance vs Indemnity
- Logistic regression model can be extended to more predictors
- Logistic regression model can include continuous variables

Multiple Logistic Regression Models: Example

```
. logistic insure1 male

------------------------------------------------------------------------------
     insure1 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        male |   1.611735   .3157844     2.44   0.015      1.09779     2.36629
------------------------------------------------------------------------------

. logistic insure2 male

------------------------------------------------------------------------------
     insure2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        male |   1.584375   .5693029     1.28   0.200     .7834322    3.204163
------------------------------------------------------------------------------
```

Multinomial Regression

- It would be convenient to have a single analysis give all the information
- Can be done with multinomial logistic regression
- Also provides more efficient estimates (narrower confidence intervals) in most cases.

Multinomial Regression Example

```
. mlogit insure male, rrr

Multinomial logistic regression                   Number of obs   =        616
                                                  LR chi2(2)      =       6.38
                                                  Prob > chi2     =     0.0413
Log likelihood = -553.40712                       Pseudo R2       =     0.0057

------------------------------------------------------------------------------
      insure |        RRR   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Prepaid      |
        male |   1.611735   .3157844     2.44   0.015      1.09779    2.36629
-------------+----------------------------------------------------------------
Uninsure     |
        male |   1.584375   .5693021     1.28   0.200     .7834329    3.20416
------------------------------------------------------------------------------
(Outcome insure==Indemnity is the comparison group)
```

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Multinomial Regression in Stata

- Command `mlogit`
- Option `rrr` (Relative risk ratio) gives odds ratios, rather than coefficients
- Option `baseoutcome` sets the baseline or reference category

Using `predict` after `mlogit`

- Can predict probability of each outcome
  - Need to give *k* variables
  - `predict p1-p3, p`
- Can predict probability of one particular outcome
  - Need to specfy which with `outcome` option
  - `predict p2, p outcome(2)`

Using `predict` after `mlogit`: Example

```
. by male: summ p1-p3

_______________________________________________________________________________
-> male = 0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          p1 |       477    .5064935           0   .5064935   .5064935
          p2 |       477    .4242424           0   .4242424   .4242424
          p3 |       477    .0692641           0   .0692641   .0692641

_______________________________________________________________________________
-> male = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          p1 |       167    .3896104           0   .3896104   .3896104
          p2 |       167     .525974           0    .525974    .525974
          p3 |       167    .0844156           0   .0844156   .0844156
```

Using `lincom` after `mlogit`

- Can use `lincom` to
  - test if coefficients are different
  - calculate odds of being in a given outcome category
- Need to specify which outcome category we are interested in
- Normally, use the option `eform` to get odds ratios, rather than coefficients

Using `lincom` after `mlogit`

```
. lincom [Prepaid]male - [Uninsure]male

 ( 1)  [Prepaid]male - [Uninsure]male = 0

------------------------------------------------------------------------------
      insure |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |    .017121   .3544299     0.05   0.961    -.6775487    .7117908
------------------------------------------------------------------------------
```

Ordinal Outcomes

- Can ignore ordering, use multinomial model
- Can use a test for trend
- Can use an ordered logistic regression model

Test for Trend

- χ^2-test tests for any differences between columns (or rows)
- Not very powerful against a linear change in proportions
- Can divide the χ^2-statistic into two parts: linear trend and variations around the linear trend.
- Test for trend more powerful against a trend
- Has no power to detect other differences
- Often used for ordinal *predictors*

Test for Trend: Example

	Treatment A		Treatment B		Total	
Healed	12	(38%)	5	(16%)	17	(27%)
Improved	10	(31%)	8	(25%)	18	(28%)
No Change	4	(13%)	8	(25%)	12	(19%)
Worse	6	(19%)	11	(34%)	17	(27%)
Total	32	(100%)	32	(100%)	34	(100%)

Test for Trend: Results

```
. ptrendi 12 5 1 \ 10 8 2 \ 4 8 3 \ 6 11 4

     +-------------------------+
     |  r   nr   _prop      x |
     |-------------------------|
  1. | 12    5   0.706   1.00 |
  2. | 10    8   0.556   2.00 |
  3. |  4    8   0.333   3.00 |
  4. |  6   11   0.353   4.00 |
     +-------------------------+

Trend analysis for proportions
------------------------------

Regression of p = r/(r+nr) on x:
Slope = -.12521, std. error =   .0546, Z =   2.293

Overall chi2(3) =          5.909,  pr>chi2 = 0.1161
Chi2(1) for trend =        5.259,  pr>chi2 = 0.0218
Chi2(2) for departure =    0.650,  pr>chi2 = 0.7226
```

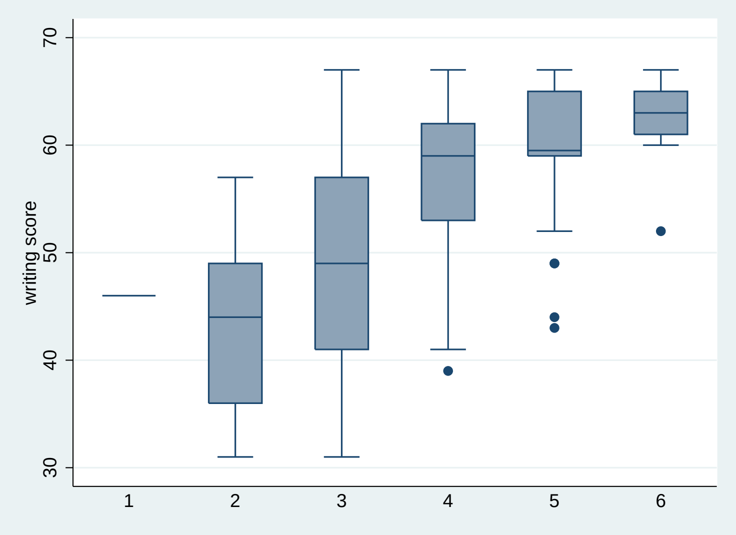
Test for Trend: Caveat

- Test for trend only tests for a linear association between predictors and outcome.
- U-shaped or inverted U-shaped associations will not be detected.
- Trend test depends on values assigned to levels of ordinal variable

Test for Trend in Stata

- Test for trend often used, should know about it
- Not implemented in base stata:
  - see http://www.stata.com/support/faqs/stat/trend.html
- Very rarely the best thing to do:
  - If trend variable is the outcome, use ordinal logistic regression
  - If trend variable is a predictor:
    - fit both categorical & continuous, `testparm` categoricals
    - if non-significant, use continuous variable
    - if significant, use categorical variables
    - Trend test, but uses appropriate regression model

Fitting an ordinal predictor



```
. regress write oread i.oread
note: 6.oread omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  5,   194) =   22.77
       Model |  6612.82672     5  1322.56534           Prob > F      =  0.0000
    Residual |  11266.0483   194  58.0724138           R-squared     =  0.3699
-------------+------------------------------           Adj R-squared =  0.3536
       Total |   17878.875   199   89.843593           Root MSE      =  7.6205

------------------------------------------------------------------------------
       write |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       oread |   3.288889   1.606548     2.05   0.042     .1203466    6.457431
             |
       oread |
          2  |  -6.669841   6.339542    -1.05   0.294    -19.17311    5.833432
          3  |  -3.666385   4.761676    -0.77   0.442    -13.05768    5.724914
          4  |   .3641026   3.568089     0.10   0.919    -6.673124    7.401329
          5  |   .4233918   2.825015     0.15   0.881    -5.148294    5.995078
          6  |          0  (omitted)
             |
       _cons |   42.71111   9.158732     4.66   0.000     24.64764    60.77458
------------------------------------------------------------------------------
. testparm i.oread

 ( 1)  2.oread = 0
 ( 2)  3.oread = 0
 ( 3)  4.oread = 0
 ( 4)  5.oread = 0

       F(  4,   194) =    1.36
            Prob > F =    0.2497
```

Dose Response

- Don't confuse trend with dose response
  - All three models may have significant trend test
  - Only first model has a dose-response effect
  - Other models better fitted using categorical variables

Genetic Model	Genotype		
	aa	aA	AA
Additive(dose-response)	0	0.1	0.2
Dominant	0	0.2	0.2
Recessive	0	0	0.2

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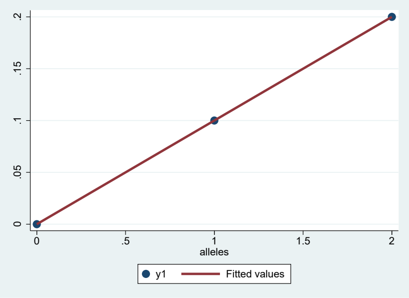

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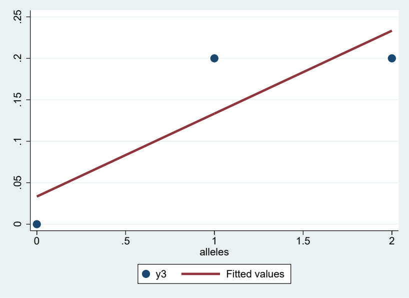

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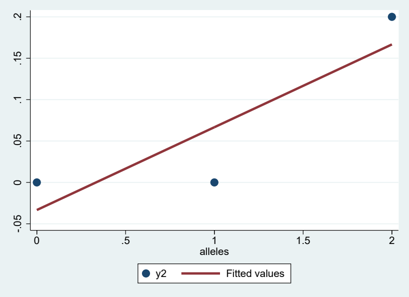

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Ordinal Regression: Using Tables

- Dichotomise outcome to “Better” or “Worse”
- Can split the table in three places
- This produces 3 odds ratios
- Suppose these three odds ratios are estimates of the same quantity
- Odds of being in a worse group rather than a better one

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$$OR_1 = \frac{(12)\times(8+8+11)}{5\times(10+4+6)} = 3.2 \quad (1)$$

$$OR_2 = \frac{(12+10)\times(8+11)}{(5+8)\times(4+6)} = 3.2 \quad (2)$$

$$OR_3 = \frac{(12+10+4)\times 11}{(5+8+8)\times 6} = 2.3 \quad (3)$$

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Ordered Polytomous Logistic Regression

$$\log\left(\frac{p_i}{1 - p_i}\right) = \alpha_i + \beta x$$

Where

- p_i = probability of being in a category up to and including the i^{th}
- α_i = Log-odds of being in a category up to and including the i^{th} if $x = 0$
- β = Log of the odds ratio for being in a category up to and including the i^{th} if $x = 1$, relative to $x = 0$
- α and p take different values for different values of i, β does not

Ordinal regression in Stata

- `ologit` fits ordinal regression models
- Option `or` gives odds ratios rather than coefficients
- Can compare likelihood to `mlogit` model to see if common odds ratio assumption is valid
- `predict` works as after `mlogit`

Ordinal Regression in Stata: Example

```
. ologit outcome treat, or

Iteration 3:   log likelihood =   -85.2492

Ordered logit estimates                           Number of obs   =         64
                                                  LR chi2(1)      =       5.49
                                                  Prob > chi2     =     0.0191
Log likelihood =   -85.2492                       Pseudo R2       =     0.0312

------------------------------------------------------------------------------
     outcome | Odds Ratio   Std. Err.      z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       treat |   2.932028   1.367427     2.31   0.021     1.175407     7.31388
------------------------------------------------------------------------------
```

Ordinal Regression Caveats

- Assumption that same β fits all outcome categories should be tested
  - AIC, BIC or LR test compared to `mlogit` model
- User-written `gologit2` can also be used
  - Allows for some variables to satisfy proportional odds, others not
  - Option `autofit()` selects variables that violate proportional odds
- There are a variety of other, less widely used, ordinal regression models: see Sander Greenland: *Alternative Models for Ordinal Logistic Regression*, Statistics in Medicine, 1994, pp1665-1677.