Future Directions in Tropical Mathematics and its Applications, 19th-20th January 2012

Mini-courses will be given by Peter Butkovic (Birmingham) and Stephane Gaubert (INRIA). Research talks will be given by Florian Block (Warwick), Alexander Guterman (Moscow), James Hook (Manchester), Katharina Huber (UEA) and Zur Izhakian (Bremen). All talks will take place in Frank Adams room 1 of the Alan Turing Building (School of Mathematics).

	Thursday 19th January		Friday 20th January
10:45	Coffee and Registration	09:00	Gaubert 2
11:30	Butkovic 1	09:45	Izhakian
12:15	Huber	10:45	Coffee
13:15	Lunch	11:15	Butkovic 3
14:15	Butkovic 2	12:15	Lunch
15:00	Block	13:15	Gaubert 3
15:45	Coffee	14:15	Coffee
16:15	Hook	14:45	Guterman
17:00	Gaubert 1	15:45	Close

Tropical linear algebra - Achievements and challenges, Peter Butkovic

The aim of the minicourse consisting of three lectures is to provide basic information on tropical linear algebra and updates on the research in some selected areas of tropical linear algebra. A number of open problems will be presented. In the first lecture basic concepts of tropical linear algebra will be introduced. These include max-linear systems, the eigenproblem and basic tools for working in this field. No prior knowledge of tropical linear algebra is assumed. In the second talk we will present an overview of recent results on reachability of eigenspaces by matrix orbits, both in the irreducible and reducible case. In the final talk we will discuss the role of tropical permanents in the analysis of a number of problems in tropical linear algebra.

From tropical convexity to ergodic control and zero-sum games, Stephane Gaubert

In this mini-lecture, I will survey the application of tropical methods to optimal control, and more generally, to zero-sum games with mean payoff. The topics covered will include: tropical convex sets and cones; tropical skew scalar product and separation theorems; extreme points and rays; nonexpansive mappings techniques; deformation of Perron-Frobenius operators (limit eigenvalues and eigenvectors and related inequalities); some combinatorial aspects of tropical convexity; equivalence between tropical convexity and zero-sum games with mean payoff; algorithmic issues. Finally, I'll discuss some infinite dimensional aspects of tropical convexity, including the Poisson-Martin type representation of stationary solutions and "abstract boundary" of control problems.

Refined Tropical Enumerative Geometry and Applications, Florian Block

According to Mikhalkin's Correspondence Theorem, complex degree-d algebraic plane curves of genus g (through sufficiently many points) are enumerated by tropical curves, counted with an integer weight. We associate a Laurent polynomial weight in q to tropical curves satisfying: (1) at q = 1, we recover the integer weights of the complex curve count, (2) at q = -1, the tropical curves compute Welschinger numbers (i.e., certain counts of real curves), and (3) the y-weighted count agrees with the recently defined "refined Severi degree" of Goettsche and Shende. This is joint work in progress with Lothar Goettsche.

Tropical matrix patterns and their applications, Alexander Guterman

The talk is based on the joint work with Ya. N. Shitov.

We introduce and investigate the notion of the tropical matrix pattern, which provides a powerful tool to investigate tropical matrices. This approach will be illustrated by the application to the investigation of the properties of the Gondran-Minoux rank function, GMr. It is known that for any tropical matrix A it holds that $trop(A) \leq GMr(A)$, where $trop(A)$ denotes the tropical rank of A. For example, applying our approach we prove that $trop(A) \geq \sqrt{GMr(A)}$ for any A. In particular as a consequence of our results we show that Gondran-Minoux rank and determinantal rank cannot be arbitrary large if the tropical rank is bounded, unlike the case with the factor rank.

Products of random max-plus matrices, James Hook

A wide variety of queuing processes give rise to Max-plus linear systems. The dynamics of such systems are dominated by a Max-plus analogue of the Lyupanov exponent which corresponds to the reciprocal of the queues throughput. The value of this exponent depends on the structure of the underlying support graph as well as the properties of the service-time distributions. For matrices whose associated weighted graphs have identically distributed edge weights we are able to decouple these two effects and provide a sandwich of analytic bounds for the Max-plus exponent, relating it to some classical properties of the support graph and extreme value expectations of the service-times. This sandwich inequality is then applied to products of componentwise exponential, Perato, Gaussian and uniform matrices. In each example we obtain an explicit relationship between the graphs greatest (classical Perron root) eigenvalue and the Max-plus exponent.

Lassoing phylogenetic trees, Katharina Huber

A classical result, fundamental to evolutionary biology, states that an edge-weighted tree T with leaf set X, positive edge weights, and no vertices of degree 2 can be uniquely reconstructed from the set of leaf-to-leaf distances between any two elements of X. In biology, X corresponds to a set of taxa (e.g. extant species) and the tree T describes their phylogenetic relationships. Provided one has access to all distances, and these are known to be sufficiently close to the distances induced by some (as yet unknown) tree, then that tree, together with its edge weighting, can be computed with some degree of confidence from those distances in polynomial time using, for example, Neighbor-Joining. However, much of the data being generated even by modern genomic methods have patchy taxon coverage whereby only certain pairs of taxa have a known (or, at least, sufficiently reliable) distance. This raises interesting mathematical questions (besides the obvious statistical and algorithmic ones) concerning tree reconstruction from such incomplete data some of which we will address in this talk.

Supertropical Algebra and Representations, Zur Izhakian

Traditionally, matroids and semigroups have been represented by using matrices defined over fields, we extend this notion by introducing new representations by matrices over semirings, more precisely over supertropical semirings. In order to bypass the lack of additive inverses in semirings, we consider a supertropical semiring – a “cover” semiring structure that has a distinguished “ghost ideal” taking the place of the zero element in many of the theorems. This supertropical structure is rich enough and permits a systematic development both of polynomial algebra and of matrix algebra, yielding direct analogs to many results and notions from classical commutative algebra. These provide a suitable algebraic framework, allowing natural representations of matroids, hereditary collections, and semigroups.