PROJECTS AND DISSERTATIONS FOR THE M.SC. IN PURE MATHEMATICS AND MATHEMATICAL LOGIC: SOME INFORMAL ADVICE

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Contents

1. Introduction

These are some informal notes and guidelines for students on the MSc Pure Mathematics and Mathematical Logic on writing MSc projects and dissertations. Much of what follows is my own personal opinion; if there is any conflict between this and what your supervisor says then go with your supervisor's opinion!

2. The project and dissertation

The MSc consists of a taught component and a dissertation component, both of which are worth 90 credits. The project and dissertation fit into this as follows:

- The project is either a 15 or 30-credit course unit undertaken during the taught component. All students have to write a project, and most students choose to write a 15-credit project. A single (15 credit) project lasts one semester and a double (30 credit) project lasts both semesters. Many students prefer to write their projects in Semester 2 (when they have a better idea of which areas of mathematics they want to specialise in). Some prefer to write their projects in the Semester 1, depending on their choice of lecture courses.

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- The dissertation is written over the summer. You should start working on your dissertation as soon as the Semester 2 exams finish; indeed, many supervisors will encourage you to do some preparatory work on the dissertation towards the end of Semester 2.

The dissertation normally involves writing a substantial account of a topic that is of current research interest; in particular, you can expect the mathematics (which you will largely be learning independently) in the dissertation to be hard! You'll also have to write the dissertation up; writing mathematics is hard (particularly if you are not used to writing essays), and writing about hard mathematics is even harder! One aim of the project is to act as a stepping-stone towards the dissertation: an opportunity to learn more about how to study mathematics independently and how to write mathematically, but on a more straightforward topic than will be covered in your dissertation.

2.1. **The project.**

A list of suggestions of project titles can be found in [Proj]. These indicate the range of topics that you could write your project on. You may be able to write your project on a topic that isn't listed, provided that there is somebody in the School who is able and willing to supervise it. Normally, lecturers are willing to supervise projects that are related either to their research interests (which you can find described on the School webpage) or to the courses that they teach. If in doubt, then the programme director will be able to offer suggestions for possible supervisors.

The level (meaning: difficulty) of mathematics covered in the project should be comparable to that in the taught lecture courses. As you will be learning the mathematics independently, as well as writing it up, you are not expected to cover the same amount of material that is covered in a lecture course; indeed, the amount of material should be approximately comparable to that which could be covered in around 7 lectures (including example sheets). In terms of the number of pages, this varies enormously depending on the style of mathematics that you are doing, how the project is typeset, etc. Normally a project is somewhere between 30–35 pages (longer if it contains lots of pictures, shorter if the mathematics has very few displayed equations). Your supervisor is the best guide to how much material you should cover.

If you are writing a double project, then the length should be approximately double that of a single project.

2.2. **The dissertation.**

The dissertation is normally an expository account of a topic of current research interest. Normally your dissertation grows out of your project (so you may find that your project becomes essentially 'Chapter 0' of your dissertation), but occasionally some students decide that they want to write their dissertations in a different area of pure mathematics; this is fine, and if you want to do this then you should discuss it with the programme director.

Your supervisor will be able to tell you the exact scope of your dissertation. A typical dissertation would be to take a suitable recent research paper and write an expository account of it in a way that would be understandable to a less-specialised audience (see §3.2 below). You would have to explain the context of the research

paper, perhaps explain some of the necessary background, and give an account of the theorems proved in the paper. You could also give some examples that illustrate the main theorems (or counter-examples to show that the hypotheses cannot be weakened). Another model for a dissertation would be to take two shorter research papers, or chapters in a graduate text or research monograph, on related topics and combine them into a coherent whole.

Note that a dissertation should normally be on a topic of current research interest. It need not contain any original research (meaning: theorems you, or you and your supervisor, have proved).

As with the project, the length of a dissertation will vary depending on the topic, the number of pictures, etc. The average length of a dissertation typeset using the University's LaTeX style file (see §3.1.2 below) is probably around 45 pages.

3. Writing

Your supervisor is the best person to ask in terms of how the project and dissertation should be written, what material should be included, what material can be assumed, etc. There are as many views on good mathematical style as there are mathematicians, and what follows is (largely) just my own personal opinion.

3.1. **Presentation.**

Your project should normally be typeset and your dissertation must be typeset. There are many different ways of typesetting mathematics. The most popular way is using a programme called LaTeX. This is free, and available for Windows, Mac OSX, Linux, etc. Other alternatives - Word, for example - exist, but many of them do not work well with the large amounts of mathematical notation that will appear in your projects and dissertations. The downside to LaTeX is that it has a steep learning curve. You are very strongly encouraged to go along to the introductory LaTeX classes that the University runs. There are also many excellent introductory guides on the Internet. Probably the best source of guidance with LaTeX is other students: the majority of PhD students (and some MSc students) are adept at using LaTeX.

Drawing diagrams and pictures in LaTeX can be cumbersome. It is perfectly fine to leave a space in the project or dissertation and then draw pictures in by hand. (To leave a 5cm vertical space in LaTeX use the command `\vspace{5cm}`.)

3.1.1. *The project.* Your project should also start with a title page, containing your name, your supervisor's name, and the fact that you are a on the MSc programme for Pure Mathematics and Mathematical Logic in Manchester. Contents pages, lists of figures, etc, can be included if you want to (particularly if they help the reader), but are not compulsory.

Your project can be bound in any way that you see fit, provided that the pages are secured in such a way that they can't easily come loose.

3.1.2. *The dissertation.* There are formatting requirements imposed by the University for the dissertation. See [PGTreg]. If you do not follow these guidelines then your dissertation may not be accepted and this will delay the publication of your result.

There is a LaTeX style file for University of Manchester theses which follow the University's formatting requirements. You can obtain this, together with a template file for your dissertation from

`https://www.maths.manchester.ac.uk/intranet/it-support/useful-files`.

3.2. **The intended audience.**

A common question is 'how much background information should I include?'. You should imagine that your target audience is another reasonably well-educated MSc student who has studied a reasonable amount of pure mathematics but who has not specialised in the areas of mathematics that you have started to specialise in. You should also remember that your examiners are looking to see whether you understand the material, so it may be helpful to include some definitions of basic concepts if they are particularly important or form a central part of your project/dissertation.

For example, in a project on nilpotent groups it would be safe to assume that the reader knows what a group is (although you may want to define some notation: will you use e, ι or 1 for the group identity?) but you should define what nilpotent means. If you are writing about ergodic theory, then you can assume the reader knows what a measure is, but you should define more advanced concepts such as topological pressure. If you are writing about algebraic topology or mathematical physics then you can assume that the reader knows what a manifold is, but it might be appropriate to define what is meant by a fibre bundle. If you write about a topic in logic you should assume that the reader knows what the completeness theorem is (after you specified the logic: propositional, predicate, modal, etc.), but you should define what is meant by an o-minimal structure, or by a recursive function $\mathbb{Q} \longrightarrow \mathbb{Q}$.

3.3. **General advice.**

Your project and dissertation will normally be an expository account of some piece of mathematics. The main sources are likely to be books, papers, or preprints—in other words, a source where the author has carefully thought about making the exposition as good as possible. How can you write your own account without just copying out what someone else has already written? Here are some suggestions:

- After stating a theorem, you could give some simple or straightforward examples to illustrate it (see §4 for an example). Or you could draw a picture to illustrate what is going on (as a simple example: if your project had the Inverse Function Theorem in n-dimensions as a major result then you could illustrate it by drawing a picture and discussing it in the 1-dimensional case $f : \mathbb{R} \to \mathbb{R}$). Remember that 'pictures are worth a thousand words'!
- Alternatively, you could give counter-examples to show that some of the hypotheses in the theorems are indeed necessary.
- If a proof involves many technical steps (this is particularly true in analysis where long strings of inequalities often occur), then you could write a paragraph that gives an overview of the main strategy of the proof.
- If a proof is particularly technical, you could give the proof (or a sketch of the proof) in a special case that doesn't require the full technical detail. You could then indicate the modifications that are necessary to make the proof work more generally.
- If a proof is essential to the topic you are writing about, you can decompose it into smaller bits, like lemmas and remarks and comment particularly on the tools used in each step. In papers, sometimes proofs are highly compressed, with many external links that are not specified in the paper. By making a sort of an exploded assembly drawing of the proof, your work can become a valuable source to penetrate the original text for subsequent readers of your manuscript.
- Many textbooks include exercises. Some of the exercises may contain interesting corollaries or relevant applications of the results you are discussing. If so, then

you could solve these problems and then include this material in your project or dissertation.

3.4. **Good mathematical style.**

There is no one correct way to write mathematics. Below is some general advice based on my own opinions (your supervisor may have different views, and in doubt you should follow their advice).

You should normally write the project in the first person plural ('We show...', 'We will prove...', 'We have seen that...', etc).

Write in complete sentences, and in proper English. Each sentence should have a subject, an object and a verb. For example,

> Let C be a perfect subset of X. And nowhere dense.

is not correct. Instead you should write

> Let C be a perfect subset of X. Suppose in addition that C is nowhere dense.

or (better)

> Let C be a perfect and nowhere dense subset of X.

You should also try to use short sentences. For example,

> Let f be a continuous function on $[0, 1]$, and so it is bounded.

is harder to read than

> Let f be a continuous function on $[0, 1]$. Then f is bounded.

Don't start sentences with mathematical symbols, and don't use mathematical notation inappropriately (particularly when it is clearer to write something in complete English). For example, don't write

> f is a continuous function on $[0, 1]$. $\exists x_0 \in [0, 1]$ s.t. $f(x_0) = \sup_{x \in X} f(x)$.

Instead write

> Let f be a continuous function on $[0, 1]$. Then there exists a point $x_0 \in [0, 1]$ such that $f(x_0) = \sup_{x \in X} f(x)$.

Your project and dissertation should be written in a more formal style than lecture notes. You should avoid verbal contractions (isn't, don't, let's) and write such expressions out in full (is not, do not, let us).

3.5. **Common mistakes.**

Here are some common typographical mistakes that many students (and people who should know better!) often make when writing mathematics.

- A sentence that ends in a displayed equation still needs a full-stop at the end. For example:

  > Consider the matrix
  > $$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$
  > This has determinant 1.

  (There is a missing full-stop after the matrix.)
- Check that the brackets match up. For example, check that you haven't written $f(T(x)$ or $f[T(x))$ when you should have written $f(T(x))$. Many text editors (such as Emacs, TeXShop or WinEdt) automatically alert you to missing brackets.
- Use apostrophes correctly.
- Spell check your work, but still check through it yourself for spelling and grammatical mistakes.

3.6. **Referencing and citation.**

The University's guide to plagiarism [Plag] states that 'plagiarism is the presenting of ideas, work or words of other people without proper, clear and unambiguous acknowledgement.' The University takes plagiarism very seriously and if you are found to have plagiarised (even inadvertently) then you can expect very serious penalties (such as a mark of zero in your project or dissertation). You should read the guide on plagiarism in the handbook carefully.

When you write your project or dissertation you should make it absolutely clear which parts are due to yourself and which parts are due to other people. If the order and structure of the material in one section in your work closely follows that in some text or paper, then you should say so. If you give a proof of a theorem that follows the proof given in some other source, then you should say so and give a reference. (One exception is that you normally need not give references to standard definitions—for example, if your project includes the definition of, say, continuity, or soluble group, or ergodicity, then it normally would not be appropriate to give a reference.) It may also be appropriate in your introduction to say what the main sources for each subsequent section of your project are. If in doubt you should ask your supervisor for guidance.

Your references should be complete and unambiguous.

- Books should be referenced by giving their author(s), title, edition (if appropriate), publisher, address of publisher (if appropriate), year of publication.
- Papers should be referenced by giving their author(s), title, journal, volume of journal, page numbers, and year of publication.
- Preprints (which are papers that have yet to appear in a journal) should give the author, the author's institution, year (and, if appropriate, arXiv webpage).
- Webpages should be cited in full with the page title, complete URL, and (for webpages whose content may change over time) the date on which that webpage was retrieved. (In particular, writing

  Prime numbers, `www.wikipedia.org`

  isn't appropriate. Instead write

  Prime numbers, `www.wikipedia.org/wiki/Primes`,
  retrieved on 15/9/2009.

There should not be any references that aren't cited somewhere in the main body of the text.

4. An example

Here is a sample of what (part of) a well-written project might look like.

> **§1.2 Continuous functions on compact spaces**
>
> Let (X, d) be a metric space. In this section we will define what is meant by a continuous function $f : X \to \mathbb{R}$. We will then prove some basic properties of such functions. Our exposition follows that in [Ru, Chapter 4].
>
> **1.2.1 Definition.** Let (X, d) be a metric space. A real-valued function $f : X \to \mathbb{R}$ is said to be <u>continuous at $x \in X$</u> if for all $\varepsilon > 0$ there exists $\delta > 0$ such that if $d(x, y) < \delta$ then $|f(x) - f(y)| < \varepsilon$.
>
> $\vdots$
>
> **1.2.2 Theorem.** *Let (X, d) be a compact metric space and let $f : X \to \mathbb{R}$ be continuous. Then $\sup_{x\in X} f(x) < \infty$. Moreover, there exists a point $x_0 \in X$ such that $f(x_0) = \sup_{x\in X} f(x)$.*
>
> *Proof.* We follow the argument in [Di, Theorem 3.17.10]. Consider the set....

⋮

This completes the proof. □

1.2.3 Remarks.

(i) Without the compactness of X the conclusion of this theorem fails in general: For example, take $X = \mathbb{R}$. Define $f : X \to \mathbb{R}$ by $f(x) = x^2$. Then X is compact, f is continuous, but clearly $\sup_{x \in X} f(x) = \infty$.

(ii) This theorem is also false if f is not continuous, even if X is compact. For example, take $X = [0, 1]$ and define $f : X \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1/x & \text{if } x \in (0, 1], \\ 0 & \text{if } x = 0. \end{cases}$$

Then f is not continuous. In this case, $\sup_{x \in X} f(x) = \infty$.

⋮

References

[Di] J. Dieudonné, Foundations of Modern Analysis, Academic Press, New York and London, 1960.

[Ru] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill, 3rd ed., 1976.

Note that I didn't give a reference for the two remarks, as I thought these examples up myself when I was writing this.

Finally, poor referencing is endemic in mathematics, particularly in lecture notes. Copying somebody else's bad practice is not an excuse! A good rule of thumb is to ask yourself 'Could someone who wasn't familiar with the area get the (mistaken) impression that this theorem or proof is due to me?'. If the answer is 'yes' then you need to improve the referencing.

5. The oral exam

The purpose of the oral exam is for the examiners to check that (i) you wrote the project yourself, and (ii) you understand what is in the project. Normally the oral exam for a single project will last around 30 minutes (and for a double project about an hour), but this may vary considerably.

Some examiners may ask you, in advance, to prepare a short presentation on your project; some examiners may just ask you questions about the material in the project. If in doubt, then you should check the format of the oral exam with your supervisor.

There is no oral exam for the dissertation.

References

[Handbook] Handbook: http://www.maths.manchester.ac.uk/study/postgraduate/information-for-current-students/informationforcurrenttaughtmscstudents/programme-handbooks/

[PGTreg] Guidance Notes for the Presentation of Taught Master's Dissertations, http://www.maths.manchester.ac.uk/media/eps/schoolofmathematics/study/postgraduate/information/Sub2Exam-FAQ-v.5.pdf 3

[Plag] Guidance to students on plagiarism and other forms of academic malpractice: http://www.staffnet.manchester.ac.uk/tlso/policy-guidance/malpractice/ 6

[Proj] The homepage of the project module: 2 http://personalpages.manchester.ac.uk/staff/Marcus.Tressl/MSc/ProjectsMATH61000-61201-61202.pdf

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