Proposition

Let $x \in \mathbb{R}$ with $x > -1$ and $n \in \mathbb{N}$. Then $(1 + x)^n \geq 1 + nx$.

Proof.

On board
Section 3.2: (Strong) mathematical induction

Let $p(n)$ be a statement about the natural number $n$.

Sometimes in the inductive step we need to assume not just that $p(k)$ is true in order to show $p(k + 1)$ is true, but also need $p(r)$ for some $r \leq k$ (could be all $r \leq k$ or could be, e.g., $k - 1$ and $k$).

Suppose we can show:

(i) $p(1)$ is true, although sometimes it is necessary to check other $p(t)$ for small $t$;

(ii) for each natural number $k$, if $p(r)$ is true for all $r \leq k$, then $p(k + 1)$ is true.

Then $p(n)$ must be true for all natural numbers $n$.

**Remark** Simple and strong induction are equivalent (see [IMR]).
Definition (Fibonacci numbers)

For each natural number $n$, define $u_n$ as follows:

\[ u_1 = 1, \quad u_2 = 1 \]
\[ u_{k+1} = u_{k-1} + u_k \] for $k \geq 2$.

These are the Fibonacci numbers. They are: 1, 1, 2, 3, 5, 8, 13, 21, 34, …

There is a beautiful non-iterative expression for the Fibonacci numbers:

**Theorem**

Let $u_n$ be the $n$th Fibonacci number, as defined in the definition. Write

\[ \alpha = \frac{1 + \sqrt{5}}{2}, \quad \beta = \frac{1 - \sqrt{5}}{2}. \]

Then $u_n = \frac{1}{\sqrt{5}} (\alpha^n - \beta^n)$.

**Proof.**

*On board*
Along the same lines, we have the following:

**Proposition**

For all natural numbers $n$, define $s_n$ by

\[
\begin{align*}
s_1 &= 4 \\
s_2 &= -1 \\
s_{n+1} &= s_n + 2s_{n-1}.
\end{align*}
\]

Then $s_n = 2^{n-1} - 3(-1)^n$ for each natural number $n$.

**Proof.**

On board