Basic idea:
Let \( p(n) \) be a statement about the natural number \( n \).
Suppose we can show:
(i) \( p(1) \) is true (this is called the base case),
(ii) for each natural number \( k \), if \( p(k) \) is true, then \( p(k + 1) \) is true (this is called the inductive step).
Then \( p(n) \) is true for all natural numbers \( n \).

**Theorem**

Let \( n \) be a natural number. Then
\[
1 + 2 + \cdots + n = \frac{n(n+1)}{2}.
\]

**Proof.**

On board
Before proceeding, we need a couple of basic facts:

**Lemma**

(i) Let $a$, $b$ and $c$ be integers and suppose $c > 0$. Then $a \leq b \Rightarrow ac \leq bc$.

(ii) Let $a$, $b$ and $n$ be natural numbers. Then $a \leq b \Rightarrow a^n \leq b^n$.

**Proposition**

Let $n$ be a natural number. Then $n! \leq n^n$.

**Proof.**

*On board*

**Class exercise**

*On board*
Theorem

Let $n$ be a natural number. Then $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$.

Proof.

On board

Proposition

Let $x \in \mathbb{R}$ with $x > -1$ and $n \in \mathbb{N}$. Then $(1 + x)^n \geq 1 + nx$.

Proof.

On board