Proposition

$\sqrt{2}$ is not a rational number.

Proof.

*On board*
Section 2 : Proof by contradiction (continued)

Proposition

Every natural number greater than one has a prime divisor.

Proof.

On board

This method of proof is often called proof by minimal counterexample. The proposition above allows us to prove a much more impressive theorem, due to Euclid:

Theorem (Euclid)

There are infinitely many prime numbers.

Proof.

On board
Proof by contrapositive
Suppose want to prove \( p \implies q \).
Saw this is equivalent to \((\neg q) \implies (\neg p)\).
So to prove \( p \implies q \) it suffices to assume \( \neg q \) and deduce \( \neg p \).

To illustrate proof by contrapositive, we apply it to a (trivial) result which it would be hard to prove otherwise:

**Proposition**

Let \( a \) and \( b \) be integers. If \( a + b \geq 9 \), then \( a \geq 5 \) or \( b \geq 5 \).

**Proof.**

On board