9 - RELATIONS

9.1 Determine whether the following relations on \( \mathbb{R} \) are (a) reflexive; (b) symmetric; (c) transitive.

(i) \( xRy \iff x \leq y + 1 \)
(ii) \( xRy \iff xy \geq 0 \)
(iii) \( xRy \iff xy > 1 \)
(iv) \( xRy \iff |x - y| \leq 1 \).

9.2 Determine which of the following relations \( R \) on the given set \( A \) are equivalence relations.

(i) \( A = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\}) \); \( (a, b)R(c, d) \iff ad = bc \).
(ii) \( A = \mathbb{Z} \); \( aRb \iff a - b \) is odd.
(iii) \( A = \mathbb{N} \); \( aRb \iff \frac{a}{b} \) is a prime number or \( \frac{a}{b} = 1 \).
(iv) \( A = \mathbb{Q} \); \( aRb \iff a - b \in \mathbb{Z} \).

9.3 Let \( f : A \to B \) be a function. Define a relation \( R \) on \( A \) by

\[ a_1Ra_2 \iff f(a_1) = f(a_2), \]

where \( a_1, a_2 \in A \).

Prove that \( R \) is an equivalence relation on \( A \).

Find the equivalence classes of \( R \) for the following functions and verify that they partition \( A \).

(i) \( A = \{1, 2, 3, 4, 5\} \), \( B = \{6, 7, 8\} \) and \( f(1) = 6 \), \( f(2) = 7 \), \( f(3) = 8 \), \( f(4) = 7 \), \( f(5) = 8 \).

(ii) \( A = B = \mathbb{Z} \); \( f(x) = (x - 1)^2 \).

(iii) \( A = \mathbb{C}, B = \mathbb{R} \); \( f(z) \) is the imaginary part of \( z \).