5.1 Which of the following are functions? In each case, if $f$ is not a function then modify the range or domain to produce a function.

(i) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \frac{1}{1+x^2}$.
(ii) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \frac{1}{1-x}$.
(iii) $f : \mathbb{Z} \to \mathbb{Z}$ with $f(x) = \frac{1}{2}x$.
(iv) $f : [-1, 1] \to \mathbb{R}$ with $f(x) = \sin^{-1} x$.
(v) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \log_e(x)$.

5.2 (9.1 of [IMR]) Determine whether each of the following functions $\mathbb{R} \to \mathbb{R}$ is 1-1, onto or bijective:

(i) $f_1(x) = 2x + 5$;
(ii) $f_2(x) = x^2 + 2x + 1$;
(iii) $f_3(x) = x^2 - 2x$;
(iv) $f_4(x) = \begin{cases} 
\frac{1}{x} & \text{if } x \neq 0 \\
0 & \text{if } x = 0 
\end{cases}$

5.3 (a) Which of the following functions are 1-1?

(i) $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = x^3$.
(ii) $g : \mathbb{R} \to \mathbb{R}$ with $g(x) = e^{-x}$.
(iii) $h : \mathbb{R} \to \mathbb{R}$ with $h(x) = x^4$.

(b) Which of the following functions are onto?

(i) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^3$.
(ii) $g : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ with $g(x) = \frac{1}{x^3}$.
(iii) $h : \mathbb{Z} \to \mathbb{Z}$ with $h(x) = x - 3$.

5.4 Let $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2 - 2|x|$ and let $g : \mathbb{R} \to \mathbb{R}$ with $g(x) = 3x - 2$. Find formulas for

(i) $f \circ f$  
(ii) $g \circ g$  
(iii) $f \circ g$  
(iv) $g \circ f$  
In each case determine the image of $-1$ under the composite function.

5.5 Let $f : A \to B$ and $g : B \to C$ be functions. Prove that

(i) if $g \circ f$ is (1-1), then $f$ is (1-1);
(ii) if $g \circ f$ is onto, then $g$ is onto.

Give an example where $g \circ f$ is both (1-1) and onto but $g$ is not (1-1) and $f$ is not onto.

5.6 Write the following permutations as products of disjoint cycles

\[
a = \left( \begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
3 & 6 & 4 & 1 & 7 & 2 & 5 
\end{array} \right)\]
Find \( a \circ b, b \circ a, b \circ c, a^{-1}, b^{-1} \) and \( c^{-1} \). Express each as a product of disjoint cycles.

5.7 Express the following permutations of \( \{1, 2, 3, 4, 5, 6, 7, 8\} \) as products of disjoint cycles:

(i) \( (124)(25)(58) \);
(ii) \( (1234)(45678) \);
(iii) \( (12)(23)(34)(45)(56)(67)(78) \).