# 3 - PROOF BY INDUCTION - EXERCISES

**3.1** (5.1 of [IMR]) Prove by induction that for all natural numbers $n$, $n^3 - n$ is divisible by 3.

**3.2** (5.2 of [IMR]) Prove by induction that $n^3 \leq 2^n$ for all integers $n$ with $n \geq 10$.

**3.3** Prove by induction on $n$ that 13 divides $2^{4n+2} + 3^{n+2}$ for all natural numbers $n$.

**3.4** Prove by induction on $n$ that $n! > 2^n$ for all natural numbers $n$ such that $n \geq 4$.

**3.5** Prove by induction on $n$ that for all natural numbers $n$

$$\sum_{j=1}^{n} j^3 = \frac{1}{4}n^2(n+1)^2.$$

**3.6** Let $(u_n)$ be the sequence of numbers defined by

$u_1 = 1$

$u_2 = 1$,

$u_{k+1} = u_{k-1} + u_k$ for $k \geq 2$

(These are the Fibonacci numbers).

Prove by induction on $n$ that

$$u_n^2 = u_{n-1}u_{n+1} + (-1)^{n-1}$$

for all natural numbers $n$ such that $n \geq 2$.

**3.7** By experimenting or otherwise, conjecture an expression for the sum of the first $n$ odd numbers and prove that it works.