2.1 (4.1 of [IMR]) Prove by contradiction that there do not exist integers $m$ and $n$ such that
\[ 14m + 21n = 100. \]

2.2 (4.2 of [IMR]) Prove by contradiction that for any integer $n$, if $n^2$ is odd, then $n$ is odd.

2.3 Prove by contradiction that there do not exist integers $m$ and $n$ such that
\[ 24n + 3m^2 = 7. \]

2.4 Let $p$ be a natural number with $p > 1$. Suppose that if $a$ and $b$ are natural numbers with $p|ab$, then $p|a$ or $p|b$. Show that $p$ must be prime.

2.5 Recall that an integer $x$ is odd if it not even. Let $x \in \mathbb{Z}$. Prove that if $x = 2a + 1$ for some $a \in \mathbb{Z}$, then $x$ is odd.
We’ll prove the other direction later.

2.6 (hard) Let $n$ be a natural number which is not a square (that is, there is no natural number $m$ such that $n = m^2$). Prove that $\sqrt{n}$ is not a rational number.
[Hint: Take inspiration from proofs in the lecture notes.]

2.7 Prove the following using the contrapositive:
(i) Let $x \in \mathbb{Z}$. If $x^3 - 4x + 3$ is even, then $x$ is odd.
(ii) Let $a, b, n \in \mathbb{Z}$. If $n \not| ab$, then $n \not| a$ and $n \not| b$. 