Morita equivalence classes of 2-blocks of defect three

Charles W. Eaton

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Abstract

We give a complete description of the Morita equivalence classes of blocks with elementary abelian defect groups of order 8 and of the derived equivalences between them. A consequence is the verification of Broué's abelian defect group conjecture for these blocks. It also completes the classification of Morita and derived equivalence classes of 2-blocks of defect at most three defined over a suitable field.

1 Introduction

Throughout let k be an algebraically closed field of prime characteristic ℓ and let $\mathcal{O}$ be a discrete valuation ring with residue field k and field of fractions K of characteristic zero. We assume that K is large enough for the groups under consideration. We consider blocks B of $\mathcal{O}G$ with defect group D.

We are concerned with the description of the Morita and derived equivalence classes of (module categories for) blocks of finite groups with a given defect group D. We briefly review progress on this problem to date. If D is an abelian p-group whose automorphism group is a p-group, then any block with defect group D must be nilpotent and so Morita equivalent to $\mathcal{O}D$ (see [14] and [22]). There are many other examples of p-groups for which it has been proved that every fusion system is nilpotent, but we do not list these here. If D is cyclic, then the Morita equivalence classes can be characterised in terms of Brauer trees, in work going back to Brauer and Dade (see [20]). In a series of papers Erdmann characterises the Morita equivalence classes of tame blocks defined over k except when D is generalised quaternion and B has two simple modules (see [8]), although the problem remains open for blocks defined over $\mathcal{O}$. The (three) Morita equivalence classes of blocks defined over $\mathcal{O}$ with defect group $C_2 \times C_2$ are determined in [19]. When $D = \langle x, y : x^{2^r} = y^{2^s} = [x,y]^2 = [x,[x,y]] = [y,[x,y]] = 1 \rangle$, where $r \geq s \geq 1$ (nonmetacylic minimal nonabelian 2-group), the Morita equivalence classes of blocks are determined in [24] and [7]. When D is a homocyclic 2-group, the Morita equivalence classes of blocks are determined in [6].

In this paper we use the classification given in [6] to completely determine the Morita and derived equivalence classes of blocks defined over $\mathcal{O}$ with defect group $D \cong C_2 \times C_2 \times C_2$. As a consequence it follows that Broué's abelian defect group conjecture holds for blocks of defect three. We also note that this completes the classification of Morita equivalence classes of 2-blocks of defect at most three, for blocks defined over k. Blocks with elementary abelian defect groups of order 8 have already been studied in [10], where it is shown that Alperin's weight conjecture and the isotypy version of Broué's abelian defect group conjecture hold for these blocks. The results of [10] are needed here, in particular to achieve Morita equivalences over $\mathcal{O}$ rather than k.

Before stating the main theorem, we recall the definition of the inertial quotient of B. Let b_D be a block of $\mathcal{O}DC_G(D)$ with Brauer correspondent B, and write $N_G(D, b_D)$ for the stabilizer in $N_G(D)$ of b_D under conjugation. Then the *inertial quotient* of B is $E = N_G(D, b_D)/DC_G(D)$, an ℓ'-group unique up to isomorphism.

Theorem 1.1 *Let B be a block of $\mathcal{O}G$, where G is a finite group. If B has defect group D isomorphic to $C_2 \times C_2 \times C_2$, then B is Morita equivalent to the principal block of precisely one of the following:*

(i) D;
(ii) $D \rtimes C_3$;
(iii) $C_2 \times A_5$, and the inertial quotient is C_3.
(iv) $D \rtimes C_7$;
(v) $SL_2(8)$, and the inertial quotient is C_7;
(vi) $D \rtimes (C_7 \rtimes C_3)$;
(vii) J_1, and the inertial quotient is $C_7 \rtimes C_3$;
(viii) ${}^2G_2(3) \cong \mathrm{Aut}(SL_2(8))$, and the inertial quotient is $C_7 \rtimes C_3$;

Blocks are derived equivalent if and only if they have the same inertial quotient.

A block with defect group $C_2 \times C_2 \times C_2$ cannot be Morita equivalent to a block with non-isomorphic defect group. This is since Morita equivalence preserves defect and (i) 2-blocks of defect three with abelian defect groups other than $C_2 \times C_2 \times C_2$ must be nilpotent (and so Morita equivalent to the group algebra of a defect group), (ii) 2-blocks of defect three with nonabelian defect groups have five irreducible characters (whilst the number is eight for blocks with defect group $C_2 \times C_2 \times C_2$).

Corollary 1.2 *Broué's abelian defect group conjecture holds for all 2-blocks with defect at most three. That is, let B be a block of $\mathcal{O}G$ for a finite group G with defect group D of order dividing 8, and let b be the unique block of $\mathcal{O}N_G(D)$ with Brauer correspondent B. Then B and b have derived equivalent module categories.*

Proof. If a defect group D are isomorphic to C_2, C_4, $C_4 \times C_2$ or C_8, then the block is nilpotent, in which case the conjecture holds automatically since $\mathrm{Aut}(D)$ is a 2-group. If $D \cong C_2 \times C_2$, then the result follows from [19]. Suppose that $D \cong C_2 \times C_2 \times C_2$. By Theorem 1.1 the derived equivalence class of B is uniquely determined by the number $l(B)$ of irreducible Brauer characters. Since every block with defect group D has eight irreducible characters, it is a consequence of Brauer's second main theorem that $l(B) = l(b)$ and the result follows. □

Note that we do not prove that there are splendid derived equivalences of blocks.

Corollary 1.3 *Let B be a block with defect group $D \cong C_2 \times C_2 \times C_2$. Then B has Loewy length $LL(B)$ equal to 4, 6 or 7.*

PROOF. By Theorem 1.1 it suffices to consider cases (i)-(viii) in the notation of that theorem. In cases (i), (ii), (iv) and (vi), where $D \lhd G$ and $[G : D]$ is odd, we have that $LL(B) = LL(kD) = 4$, by [12, 4.1]. In case (iii) $LL(B) = 6$. In the remaining cases $LL(B) = 7$ by [1] and [18], again using [12, 4.1]. □

Corollary 1.4 *Let B be a 2-block of defect at most 3, then the Cartan invariants of B are at most the order of a defect group.*

Of course the above does not hold in generality.

Since we now have a complete list of Cartan matrices (up to ordering of the simple modules), and indeed the decomposition matrices, for 2-blocks of defect at most 3, it would be interesting to look for possible concrete restrictions on Cartan matrices.

2 Quoted results

The following proposition will be used when considering automorphism groups of simple groups. It gathers together two propositions from [10], which in turn gathers results from [5] and [15].

Proposition 2.1 *Let ℓ be any prime and let G be a finite group and $N \lhd G$ with $[G : N] = w$ a prime not equal to ℓ. Let b be a G-stable ℓ-block of $\mathcal{O}N$. Then either each block of $\mathcal{O}G$ covering b is Morita equivalent to b, or there is a unique block of $\mathcal{O}G$ covering b. In the former case, B and b have isomorphic inertial quotient.*

PROOF. Note that the group $G[b]$ of elements of G acting as inner automorphisms on b is a normal subgroup of G containing N. If $G[b] = G$, then each block of G covering b is source algebra equivalent to b by [10, 2.2], and has inertial quotient isomorphic to that of b by [10, 3.4]. If $G[b] = N$, then there is a unique block of G covering b by [10, 2.3]. □

The following is a distillation of those results in [16] which are relevant here.

Proposition 2.2 ([16]) *Let G be a finite group and $N \lhd G$. Let B be a block of $\mathcal{O}G$ with defect group D covering a G-stable nilpotent block b of $\mathcal{O}N$ with defect group $D \cap N$. Then there is a finite group L and $M \lhd L$ such that (i) $M \cong D \cap N$, (ii) $L/M \cong G/N$, (iii) there is a subgroup D_L of L with $D_L \cong D$ and $D_L \cap M \cong D \cap N$, and (iv) there is a a central extension $\tilde{L}$ of L by an ℓ'-group, and a block $\tilde{B}$ of $\mathcal{O}\tilde{L}$ which is Morita equivalent to B and has defect group $\tilde{D} \cong D$.*

Proposition 2.3 ([26]) *Let B be an ℓ-block of $\mathcal{O}G$ for a finite group G and let $Z \leq O_\ell(Z(G))$. Let $\bar{B}$ be the unique block of $\mathcal{O}(G/Z)$ corresponding to B. Then B is nilpotent if and only if $\bar{B}$ is nilpotent.*

Proposition 2.4 ([6]) *Let B be a block of $\mathcal{O}G$ for a quasisimple group G with elementary abelian defect group D of order 8. Then one of the following occurs:*

(i) $G \cong SL_2(8)$ and B is the principal block;

(ii) $G \cong {}^2G_2(q)$*, where* $q = 3^{2m+1}$ *for some* $m \in \mathbb{N}$*, and* B *is the principal block;*
(iii) $G \cong J_1$ *and* B *is the principal block;*
(iv) $G \cong Co_3$ *and* B *is the unique non-principal* 2*-block of defect* 3*;*
(v) G *is of type* $D_n(q)$ *or* $E_7(q)$ *for some* q *of odd prime power order,* $O_2(G) = 1$ *and* B *is Morita equivalent to the principal block of* $C_2 \times A_5$ *or of* $C_2 \times A_4$*.*
(vi) $|O_2(G)| = 2$ *and* $D/O_2(G)$ *is a Klein four group;*
(vii) B *is nilpotent.*

Lemma 2.5 *Let* B *be a block of* $\mathcal{O}G$ *for a finite group* G *with normal defect group* $D \cong C_2 \times C_2 \times C_2$*. Then* B *is Morita equivalent to* $\mathcal{O}(D \rtimes E)$*, where* E *has odd order and acts faithfully on* D*.*

Proof. This is well known, but may be obtained for instance by applying Proposition 2.2 and noting that the inertial quotient is one of 1, C_3, C_7 and $C_7 \rtimes C_3$, each having trivial Schur multiplier. □

3 Preliminary results

Proposition 3.1 *Let* $N = {}^2G_2(q)$*, where* $q = 3^{2m+1}$ *for some* $m \in \mathbb{N} \cup \{0\}$*, and* $N \leq G \leq \mathrm{Aut}(N)$*. Let* b *be the principal* 2*-block of* $\mathcal{O}N$*. Then every block of* $\mathcal{O}G$ *covering* b *is source algebra equivalent to* b*. Further, each of these blocks shares a defect group with* b *and has isomorphic inertial quotient.*

Proof. G/N is cyclic of odd order. Let $N = G_0 \leq G_1 \leq \cdots \leq G_n = G$, with each $|G_{i+1}/G_i|$ prime. By [25] b has defect groups of the form $C_2 \times C_2 \times C_2$ and irreducible character degrees occurring with multiplicity either one or two, so that each irreducible character is G-stable. Since $[G : N]$ is odd each block of $\mathcal{O}G_i$ covering b shares a defect group with b. By [10], every block with defect group $C_2 \times C_2 \times C_2$ (in particular b and every block of $\mathcal{O}G_i$ covering it) has precisely eight irreducible characters, and it follows that for each i there are $[G_{i+1} : N]$ 2-blocks of $\mathcal{O}G_{i+1}$ covering b, and amongst these there $[G_{i+1} : G_i]$ blocks of $\mathcal{O}G_{i+1}$ covering each such block of $\mathcal{O}G_i$. It follows from Proposition 2.1 that each block of $\mathcal{O}G_i$ covering b is source algebra equivalent to b. That the blocks have isomorphic inertial quotient follows from [10, 3.4]. □

Proposition 3.2 *Let* G *be a finite group and* $N \lhd G$ *with* $[G : N]$ *an odd prime. Let* b *be a* G*-stable block of* $\mathcal{O}N$ *with defect group* $C_2 \times C_2 \times C_2$ *and inertial quotient* C_3*. Suppose that* $l(b) = 3$*. Let* B *be a block of* $\mathcal{O}G$ *covering* b*. Then either* B *is source algebra equivalent to* b *or nilpotent. In the former case* B *has inertial quotient* C_3 *and* $[G : N] = 3$*.*

Proof. By [10] we have $l(B) \leq 7$. Suppose first that $[G : N] \geq 5$. Since we are assuming that $l(b) = 3$, there cannot be a unique block of $\mathcal{O}G$ covering b (since each irreducible Brauer character of b is G-stable and so the total number of irreducible Brauer characters in blocks covering B is at least 15), so by Proposition 2.1 B is source algebra equivalent to b and has the same inertial quotient.

Suppose now that $[G : N] = 3$. If every irreducible Brauer character of b is G-stable, in which case again by Proposition 2.1 B is source algebra equivalent to b and has the same inertial quotient. If the three irreducible Brauer characters are permuted transitively, then $l(B) = 1$, so that by [10] B is nilpotent. $\square$

The following is a strengthening of a special case of the main result of [11], which is only known to hold for blocks defined over k.

Proposition 3.3 *Let G be a finite group and $N \lhd G$ and let C be a G-stable block of $\mathcal{O}N$ covered by a block B of $\mathcal{O}G$ with elementary abelian defect group D of order 8. Write $P = N \cap D$ and suppose that $D = P \times Q$ for some Q of order 2 such that $G = N \rtimes Q$. Then $B \cong C \otimes_{\mathcal{O}} \mathcal{O}Q$. In particular B and the block $C \otimes_{\mathcal{O}} \mathcal{O}Q$ of $\mathcal{O}(N \times Q)$ are Morita equivalent.*

PROOF. Write $Q = \langle x \rangle$. As noted in [11] B and C share a block idempotent e, so that B is a crossed product of C with Q and it suffices to find a graded unit of $Z(B)$ of degree x and order two. We do this by exploiting the existence of a perfect isometry as shown in [10, 5.1], although we must show that this perfect isometry satisfies additional properties. Part of the proof follows that of [10, 5.1], and we take facts from there without explicit further reference. Note however that for convenience we use a different labeling of the irreducible characters.

Denote by E the inertial quotient of B, so that $|E| = 1$ or 3. If $|E| = 1$, then B is nilpotent and the result follows from [22]. Hence we may assume that $|E| = 3$ and E acts faithfully on D. Write $H = D \rtimes E$. Then $Q \leq Z(H)$ and so $H = (P \rtimes E) \times Q$.

By [17] we have $k(B) = 8$. Label the irreducible characters θ_i of H so that $\theta_1, \dots, \theta_4$ have Q in their kernel, $\theta_1(1) = \theta_2(1) = \theta_3(1) = 1$, $\theta_4(1) = 3$ and $\theta_i(g) = \theta_{i-4}(g)$ for all $i = 5, \dots, 8$ and all $g \in P \rtimes E$. We have $\theta_i(x) = -\theta_i(1)$ for $i = 5, \dots, 8$. Similarly label the irreducible characters $\chi_1, \dots, \chi_8$ of B so that $\mathrm{Res}^G_N(\chi_i) = \mathrm{Res}^G_N(\chi_{i-4})$ for all $i = 5, \dots, 8$. Note that $\chi_i(x) = -\chi_{i-4}(x)$ for all $i = 5, \dots, 8$.

There is a stable equivalence of Morita type between $\mathcal{O}H$ and B, leading to an isometry $L^0(H, \mathcal{O}H) \cong L^0(G, B)$ between the groups of generalised characters vanishing on 2-regular elements. $L^0(H, \mathcal{O}H)$ is generated by

$$\{\theta_1 - \theta_5, \theta_2 - \theta_6, \theta_3 - \theta_7, \theta_4 - \theta_8, \theta_1 + \theta_2 + \theta_3 - \theta_4\}.$$

We claim that if $\chi_i - \chi_j \in L^0(G, B)$, then $|i - j| = 4$. For suppose that $\chi_i(g) = \chi_j(g)$ for all $g \in G$ of odd order. Then $\mathrm{Res}^G_N(\chi_i)$ and $\mathrm{Res}^G_N(\chi_2)$ are irreducible characters of C agreeing on 2-regular elements. Noting that C is not nilpotent, and that C has decomposition matrix that of the principal 2-block of A_4 or A_5, it follows that $\mathrm{Res}^G_N(\chi_i) = \mathrm{Res}^G_N(\chi_2)$ and the claim follows.

Hence the isometry takes elements of the form $\theta_i - \theta_{i-4}$ to elements of the form $\delta_j(\chi_j - \chi_{j-4})$. Now the isometry extends to a perfect isometry $I : \mathbb{Z}\,\mathrm{Irr}(H) \to \mathbb{Z}\,\mathrm{Irr}(B)$, and we have seen that $I(\theta_i)(g) = I(\theta_{i-4})(g)$ for every $i = 5, \dots, 8$ and every $g \in N$.

Following [3] I induces an $\mathcal{O}$-algebra isomorphism $I^0 : Z(\mathcal{O}H) \to Z(B)$ with $I^0(x) = \frac{1}{|H|} \sum_{g \in G} \mu(g^{-1}, x) g$, where $\mu(g, h) = \sum_{i=1}^{8} \theta_i(h) I(\theta_i)(g)$ for $g \in G$ and $h \in H$. We will show that $I^0(x) = ax$ for some $a \in \mathcal{O}N$, i.e., that $\mu(g, x) = 0$ whenever $g \in N$. Then $I^0(x)$ will be the required graded unit of $Z(B)$ of degree x and order two.

Let $g \in N$. Then

$$\mu(g,x) = \sum_{i=1}^{8} \theta_i(x) I(\theta_i)(g) = \sum_{i=5}^{8} \theta_{i-4}(1) \left(I(\theta_{i-4})(g) - I(\theta_i)(g) \right) = 0$$

and we are done. □

4 Proof of the main theorem

We prove Theorem 1.1.

Proof. Let B be a block of $\mathcal{O}G$ for a finite group G with defect group $D \cong C_2 \times C_2 \times C_2$ with $[G : Z(G)]$ minimised such that B is not Morita equivalent to any of (i)-(viii). By minimality and the first Fong reduction B is quasiprimitive, that is, for every $N \lhd G$ each block of $\mathcal{O}N$ covered by B is G-stable. By Proposition 2.2 if $N \lhd G$ and B covers a nilpotent block of $\mathcal{O}N$, then $N \leq Z(G)O_2(G)$. In particular $O_{2'}(G) \leq Z(G)$

Following [2] write $E(G)$ for the *layer* of G, that is, the central product of the subnormal quasisimple subgroups of G (the *components*). Write $F(G)$ for the Fitting subgroup, which in our case is $F(G) = Z(G)O_2(G)$. Write $F^*(G) = F(G)E(G) \lhd G$, the generalised Fitting subgroup, and note that $C_G(F^*(G)) \leq F^*(G)$. Let b be the (unique) block of $\mathcal{O}F^*(G)$ covered by B.

Let $\overline{B}$ be the unique block of $\mathcal{O}(G/O_2(Z(G)))$ corresponding to B. First observe that $|O_2(Z(G))| \leq 2$, for otherwise $\overline{B}$ would have defect at most one and so would be nilpotent, which in turn would mean that B would be nilpotent by Proposition 2.3, a contradiction.

If $|O_2(G)| > 4$, then $O_2(G) = D$, a contradiction by Lemma 2.5. Hence $|O_2(G)| \leq 4$.

Claim. $O_2(G) \leq Z(G)$ and $|O_2(G)| \leq 2$.

Suppose that $O_2(G) \not\leq Z(G)$ (so $|O_2(G)| = 4$). If $O_2(Z(G)) \neq 1$, then $O_2(G/O_2(Z(G)))$ has order 2 and so is central in $G/O_2(Z(G))$, from which it follows using Proposition 2.3 that $\overline{B}$, and so B, is nilpotent, again a contradiction. If $O_2(Z(G)) = 1$, then $F^*(G) = O_2(G) \times (Z(G)E(G))$. Since $|O_2(G)| = 4$, B covers a nilpotent block of $F^*(G)$ and so $F^*(G) = O_2(G)Z(G)$. But $C_G(F^*(G)) \leq F^*(G)$ and so $D \leq C_G(O_2(G)) \leq O_2(G)Z(G)$, a contradiction. Hence $O_2(G) \leq Z(G)$ (and $|O_2(G)| \leq 2$) as claimed.

Write $E(G) = L_1 * \cdots * L_t$, where each L_i is a component of G (arguing as above we have that $t \geq 1$). Now B covers a block b_E of $\mathcal{O}E(G)$ with defect group contained in D, and b_E covers a block b_i of $\mathcal{O}L_i$. Since b_E is G-stable, for each i either $L_i \lhd G$ or L_i is in a G-orbit in which each corresponding b_i is isomorphic (with equal defect). Since B has defect three, it follows that if $t > 1$, then B covers a nilpotent block of a normal subgroup generated by components of G, a contradiction. Hence $t = 1$. So G has a unique component L_1, and $G/Z(G) \leq \mathrm{Aut}(L_1Z(G)/Z(G))$.

Suppose that $O_2(G) \not\leq [L_1, L_1]$. Then $F^*(G) = O_2(G) \times Z(G)L_1$. In this case $D \leq F^*(G)$, since otherwise b would be nilpotent. Since b is G-stable, this means $[G : F^*(G)]$ odd and so $O_2(G)$ is in fact a direct factor of G. By [19] it follows that B is Morita equivalent to one of (ii) or (iii), a contradiction. Hence $O_2(G) \leq [L_1, L_1]$.

We next show that $D \leq F^*(G)$. Suppose otherwise. Then since D is elementary abelian we may write $D = (D \cap F^*(G)) \times Q$ for some Q of order 2 (if Q were to be larger, then B would cover a nilpotent block of $\mathcal{O}F^*(G)$). By the Schreier conjecture $G/F^*(G)$ is solvable. Since b is G-stable, $DF^*(G)/F^*(G)$ is a Sylow 2-subgroup of $G/F^*(G)$. Hence $G = H \rtimes Q$ for some $H \lhd G$. By Proposition 3.3 $B \cong b \otimes_{\mathcal{O}} \mathcal{O}Q$ as $\mathcal{O}$-algebras. Now $b \otimes_{\mathcal{O}} \mathcal{O}Q$ is a block of $\mathcal{O}(H \times Q)$ with defect group $D = (D \cap H) \times Q$. Since b is Morita equivalent to the principal block of $\mathcal{O}A_4$ or $\mathcal{O}A_5$, it follows that B is Morita equivalent to one of (ii) or (iii). Hence $D \leq F^*(G)$. Since $[F^*(G) : L_1]$ is odd, this means D is also a defect group for b_1.

We now refer to Proposition 2.4. Suppose that $L_1 \cong SL_2(8)$ and b_1 is the principal block. Then G is $SL_2(8)$ or $\mathrm{Aut}(SL_2(8)) \cong SL_2(8) \rtimes C_3 \cong {}^2G_2(3)$, leading to (v) or (viii) of the theorem.

If $L_1 \cong {}^2G_2(3^{2m+1})$ for some $m \in \mathbb{N}$, then $L_1 \leq G \leq \mathrm{Aut}({}^2G_2(3^{2m+1}))$ and by Proposition 3.1 B is Morita equivalent to b_1. By [21, Example 3.3] b_1 is Morita equivalent to the principal block of $\mathcal{O}({}^2G_2(3))$.

If $L_1 \cong J_1$ or Co_3, then $G = L_1$. By [13, 1.5] the 2-block of $\mathcal{O}Co_3$ of defect three is Morita equivalent to the principal block of $\mathcal{O}({}^2G_2(3))$, hence we are done in this case. The principal block of $\mathcal{O}J_1$ is not Morita equivalent to that of $\mathcal{O}({}^2G_2(3))$, since their decomposition matrices are not similar (see [9] for decomposition matrices).

Suppose that L_1 is of type $D_n(q)$ or $E_7(q)$ and b_1 is Morita equivalent to the principal block of $\mathcal{O}(C_2 \times A_4)$ or $\mathcal{O}(C_2 \times A_5)$. Then G/L_1 is abelian and of odd order. By Proposition 3.2 B is either nilpotent (a contradiction) or Morita equivalent to b_1, and we are done in this case.

This leaves the case that $|O_2(L_1)| = 2$ and $D/O_2(L_1)$ is a Klein four group. We have shown that $O_2(N) = O_2(G)$. Recall that $\overline{B}$ is the unique block of $\mathcal{O}(G/O_2(G))$ corresponding B, and note that $\overline{B}$ has defect group $D/O_2(G)$. By[4] $\overline{B}$ is source algebra equivalent to the principal block of $\mathcal{O}A_4$ or of $\mathcal{O}A_5$. It follows from [23, Corollary 1.14] that B is Morita equivalent to the principal block of a central extension of A_4 or A_5 by a group of order 2, i.e., of $C_2 \times A_4$ or $C_2 \times A_5$.

To see that the blocks in cases (i)-(viii) represent distinct Morita equivalence classes it suffices to note that they have distinct decomposition matrices.

□

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Charles Eaton
School of Mathematics
University of Manchester
Oxford Road
Manchester
M13 9PL
United Kingdom
charles.eaton@manchester.ac.uk