(Relative) singularity categories are triangulated categories associated with (non-commutative resolutions of) singular varieties. I will explain these notions and their mutual relations focusing on the simplest examples - the singularities of type $A_1$, e.g. $k[x]/x^2$. For these examples, everything can be understood in a rather elementary way. In particular, familiarity with triangulated categories will NOT be necessary to follow the talk. In the end, I will mention what we know for ADE-singularities in general. This is based on joint work with Dong Yang.