10.1 (i) 2^2 \cdot 3^2; (ii) 3.13; (iii) 2^2 \cdot 5^2; (iv) 17^2; (v) 2.3.37; (vi) 2^8; (vii) 5.103; (viii) 23.43; (ix) 2^4 \cdot 3^2 \cdot 5^7; (x) 2^6 \cdot 5^3; (xi) 3.5.7 \cdot 13; (xii) 3^2 \cdot 11 \cdot 101; (xiii) 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19.

10.2 The positive integers dividing \( N = 2^{n-1}p \) are

\[ 1, 2^1, 2^2, \ldots, 2^n - 1, p, 2^1 p, 2^2 p, \ldots, 2^{n-2} p, N \]

\[ = \{ 2^i p^j : i \in \mathbb{Z}, 0 \leq i \leq n - 1, j = 0, 1 \}. \]

Adding all such divisors gives

\[ 2^0 + 2^1 + \cdots + 2^{n-1} + 2^0 p + 2^1 p + \cdots + 2^{n-1} p \]

\[ = (2^0 + 2^1 + \cdots + 2^{n-1}) + (2^0 + 2^1 + \cdots + 2^{n-1})p \]

\[ = (2^n - 1) + (2^n - 1)p = (2^n - 1)(1 + p) = p2^n = 2N. \]

Hence the sum of all divisors excluding \( N \) is \( N \).

Four prime numbers \( p \) with \( p = 2^n - 1 \) for some \( n \) are 3 = 2^2 - 1, 7 = 2^3 - 1, 31 = 2^5 - 1 and 127 = 2^7 - 1. Hence 6 = 2 \cdot 3, 28 = 2^2 \cdot 7, 496 = 2^4 \cdot 31 and 8128 = 2^6 \cdot 127 are perfect numbers.

10.3 (a) \( 11^{650} = 11^{(108 \times 6)+4} = (11^6)^{108} \cdot 11^2 \). By FMT, \( 11^6 \equiv 1 \mod 7 \), so \( 11^{650} \equiv 11^2 \equiv 121 \equiv 2 \mod 7 \).

(b) \( 6^{178} = 6^{(16 \times 11)+2} = (6^{16})^{11} \cdot 6^2 \equiv 36 \equiv 2 \).