

Noncommutative Projective Surfaces

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One of the outstanding problems of noncommutative projective geometry is to classify all noncommutative projective surfaces and in this talk we will describe a solution to this problem for a large class of algebras. This work is joint with Dan Rogalski.

Let $A = k + A_1 + A_2 + \dots$ be a graded, noetherian k -algebra that is generated in degree one over an algebraically closed field k . A useful intuition is to think of $qgr A$, the category of noetherian graded A -modules modulo those of finite length, as the category of coherent sheaves on the (nonexistent) noncommutative scheme $Proj(A)$.

Suppose that the graded quotient ring $Q(A)$ has the form $Q(A) = k(X)[t, t^{-1}, \sigma]$, where σ is an automorphism of the irreducible projective variety X ; in this case $Proj(A)$ would be the noncommutative projective surface of the title. Then we prove that A can be written as a so-called naive blowup algebra of a projective surface Y birational to X . This enables one to obtain a deep understanding of the structure both of A and of $qgr A$. In some ways, $qgr A$ is just like the category of coherent sheaves on Y , but some of its properties are strange.